Princess Nora University
Faculty of Computer & Information Systems

جامعة الأميرة نورة بنت عبد الرحمن
Princess Nora Bint Abdul Rahman University

ARTIFICIAL INTELLIGENCE

(CS 370D)

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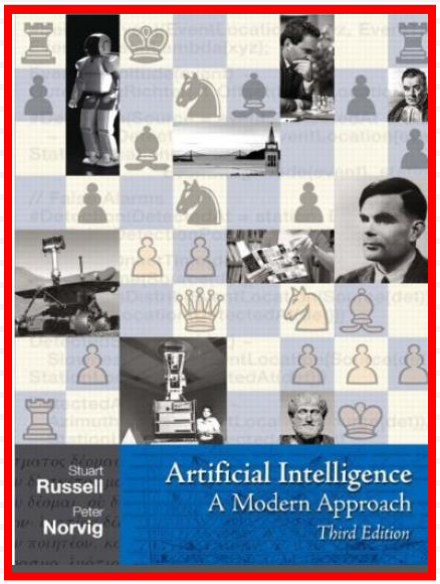

(CHAPTER-6)
CONSTRAINT SATISFACTION PROBLEMS

Outline

- What is a CSP
- CSP applications
- Backtracking search
- Problem structure and decomposition

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Mean of Constraint Satisfaction Problems

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Constraint Satisfaction Problems

- A constraint satisfaction problem consists of three components,

  X, D, and C

X is a set of variables, **{Xi, . , XT,}.**

D is a set of domains, {Di,…, Dn.}, one for each variable.

C is a set of constraints that specify allowable values.

  - Each constraint C_i limits the values that variables can take,
    - e.g., $V_1 \neq V_2$

A *state* is defined as an *assignment* of values to some or all variables.

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CSPs (continued)

- An assignment is *complete* when every variable is mentioned.
- A *solution* to a CSP is a complete assignment that satisfies all constraints.
- Some CSPs require a solution that maximizes an *objective function*.

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Varieties of constraints

- Unary constraints involve a single variable.
  - e.g. *SA* ≠ *green*
- Binary constraints involve pairs of variables.
  - e.g. *SA* ≠ *WA*
- Higher-order constraints involve 3 or more variables.
  - Professors A, B,and C cannot be on a committee together
  - Can always be represented by multiple binary constraints
- Preference (soft constraints)
  - e.g. *red* is better than *green* often can be represented by a cost for each variable assignment
  - combination of optimization with CSPs

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Constraint Satisfaction Applications

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CSPs (continued) Examples of Applications

- Scheduling :
  - Scheduling the timetable of lectures
  - Airline schedules
  - Scheduling your MS or PhD thesis exam ☺
- Configuration
- To arrange objects of different sizes in a container, Map Coloring.
- CSP solvers can be faster than state-space searchers because the CSP solver can quickly eliminate large swatches of the search space.

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Example problem – timetable of university (1)

- state - collection of schedule objects
  - schedule
    - professor(~10^2) (~350 at LU)
    - course (~10^3)
    - time slot (~10) (16 at LU)
    - location (~10^2)
- constraints
- objective function

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Example problem – timetable of university (1)

- **Constraints**
  - professors assigned to courses
  - one course per location per time slot
  - location capacity >= course (projected) enrollment
  - professors assigned only once per time slot
  - rooms assigned only once per time slot
  - courses in program in different time slots

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Example problem – timetable of university (1)

- **objective function**

  - minimize total course enrollment in early morning time slots
  - distribute program courses across days
  - accommodate special requests

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Solving CSPs

- analysis of problem state structure is critical
- two basic approaches:
  - start state is null: assign state variables one at a time and backtrack if constraint is violated
  - assign variables 'randomly/intelligently' to make a start state and use greedy search by changing variable assignments

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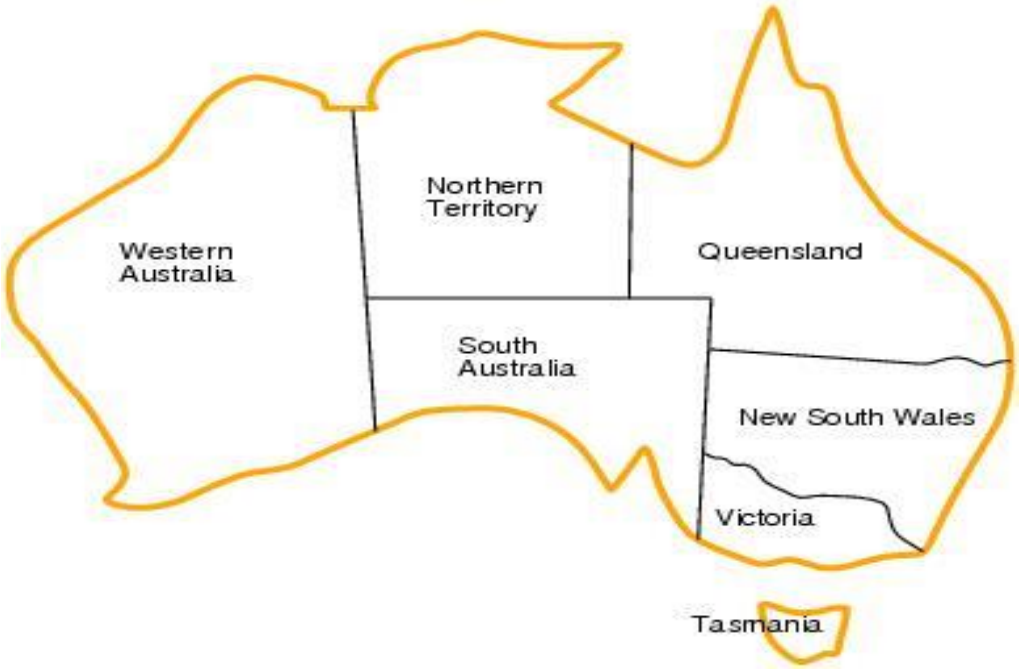
CSP example: map coloring



- Variables: *WA, NT, Q, NSW, V, SA, T*
- Domains: D_i={*red,green,blue*}
- Constraints: adjacent regions must have different colors.
  - E.g. *WA* ≠ *NT*

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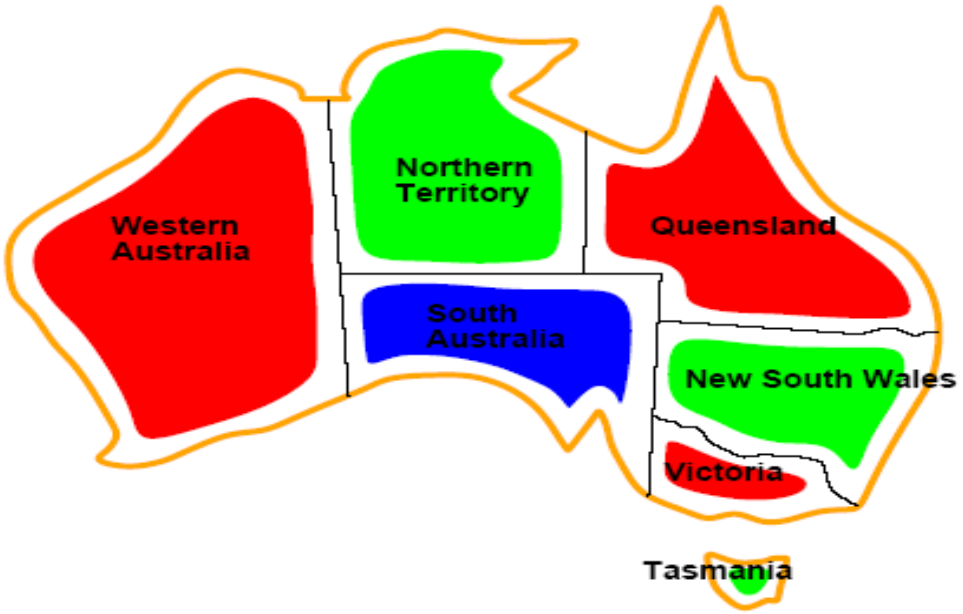
CSP example: map coloring



- Solutions are assignments satisfying all constraints, e.g.

{*WA=red,NT=green,Q=red,NSW=green,V=red,SA=blue,T=green*}

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Solving CSPs

- As general searching problem.
- Back-Tracking.
- Arc-consistency Checking.
- Heuristics.

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As General Searching Problem

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CSP as a standard search problem

- A CSP can easily be expressed as a standard search problem.
- **Incremental formulation**
  - ***Initial State***: the empty assignment {}
  - ***Successor function***: Assign a value to any unassigned variable provided that it does not violate a constraint
  - ***Goal test***: the current assignment is complete
    (by construction its consistent)
  - *Path cost*: constant cost for every step (not really relevant)

Backtracking Search

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Backtracking Search

- Variable assignments are commutative if the order of any given set of actions has no effect on the outcome.

  i.e., [WA = red then NT = green] same as [NT = green then WA = red].

- Only need to consider assignments to a single variable at each node. and backtracks when a variable has no legal values left to assign.
- Depth-first search for CSPs with single-variable assignments is called **Backtracking Search**:

- ✓ Chooses values for one variable at a time.
- ✓ Checks for constraint violations at each assignment.

- ✓ Backtracks when a variable has no legal values left to ass

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Backtracking search

function BACKTRACKING-SEARCH(*csp*) **return** a solution or failure

return RECURSIVE-BACKTRACKING({} , *csp*)

function RECURSIVE-BACKTRACKING(*assignment*, *csp*) **return** a solution or failure

if *assignment* is complete **then return** *assignment*

var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[*csp*],*assignment*,*csp*)

for each *value* **in** ORDER-DOMAIN-VALUES(*var, assignment, csp*) **do**

if *value* is consistent with *assignment* according to CONSTRAINTS [*csp*]

then

add {*var*=*value*} to assignment

result ← RRECURSIVE-BACTRACKING(*assignment, csp*)

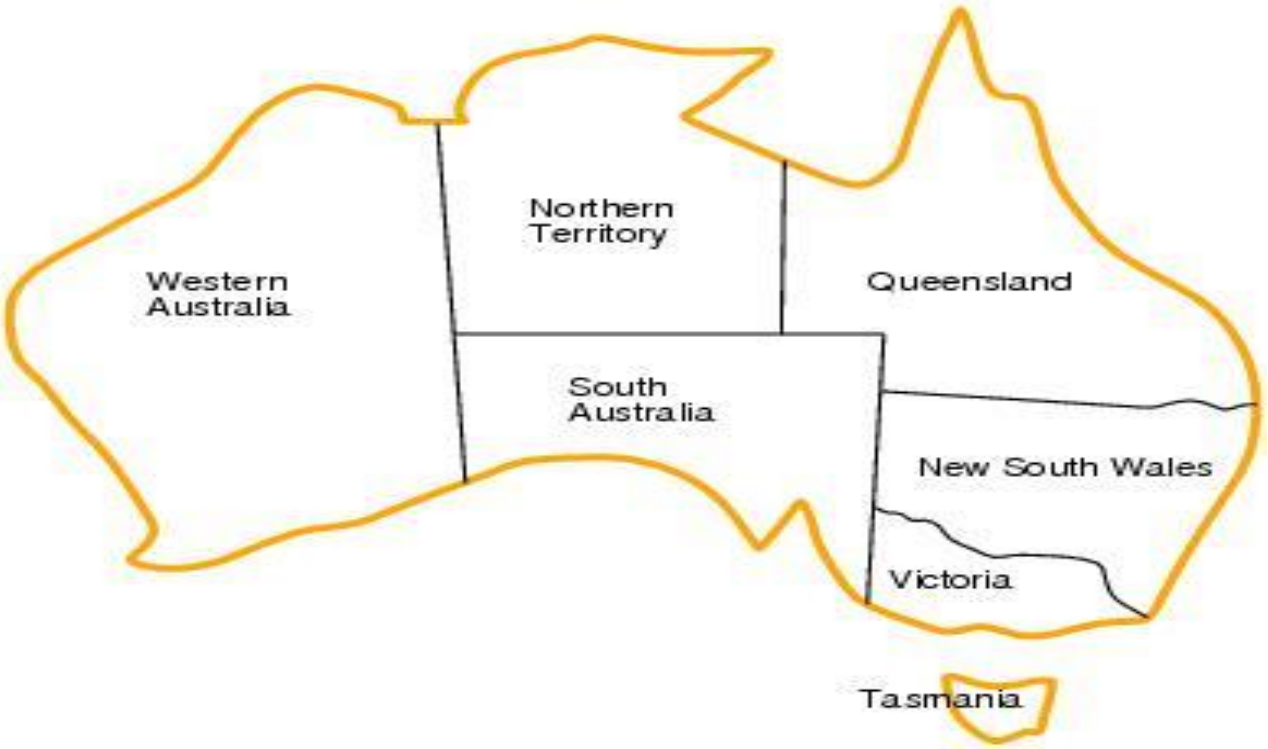
if *result* ≠ *failure* **then return** *result*

remove {*var*=*value*} from *assignment*

return *failure*

Backtacking

Backtracking example



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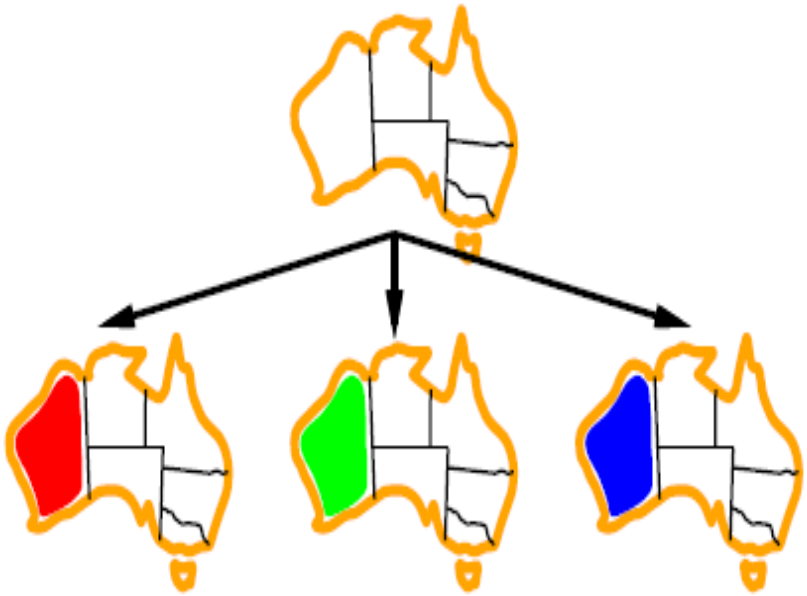
Backtracking example



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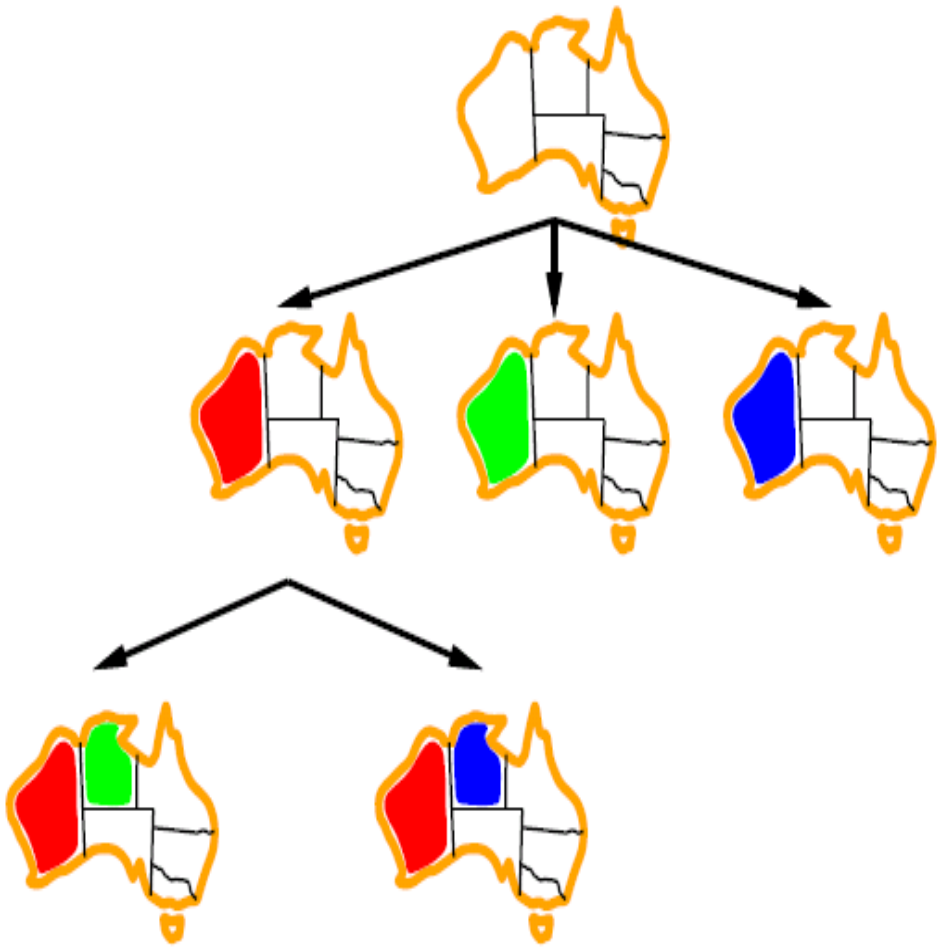
Backtracking example



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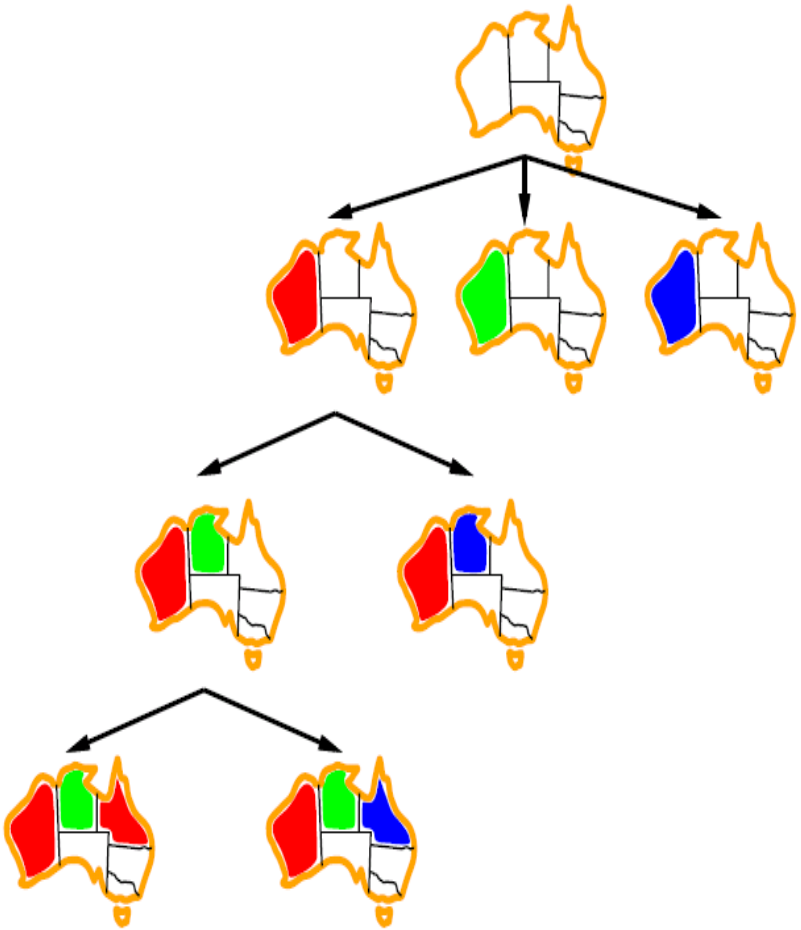
Backtracking example



Constraints on SA will eventually cause failure when WA != Q. When not the same color (bottom right), SA cannot be assigned.

The algorithm will backtrack to a node with unexplored states.

For example, **such as WA=red, NT=blue.**

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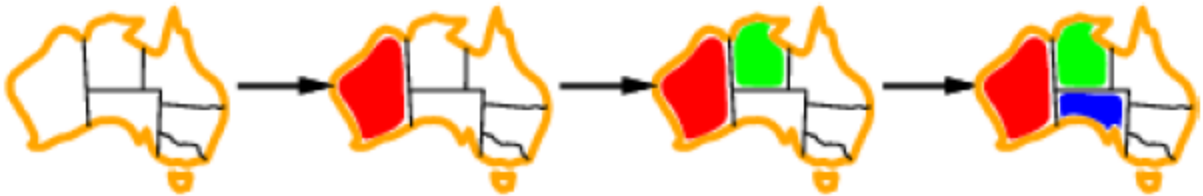
Improving Backtracking Efficiency

- **Consider**
  - Which variable should be assigned next?
  - In what order should the variables be tried?
  - Can we detect inevitable failure early?
  - Can we take advantage of problem structure?

Most Constrained Variable or minimum remaining values (MRV)

- Heuristic Rule: Choose the variable with the **fewest** legal values.
- **Idea** is that variable with *fewest* choices will likely fail sooner allowing branch to be *pruned* early.
  - Legal values are those that do not conflict with any already assigned to a neighbor (i.e. do not violate constrains).
  - After WA=red, NT and SA have only 2 remaining legal values, the other unassigned variables all have 3.
  - **Choose between NT and SA.**



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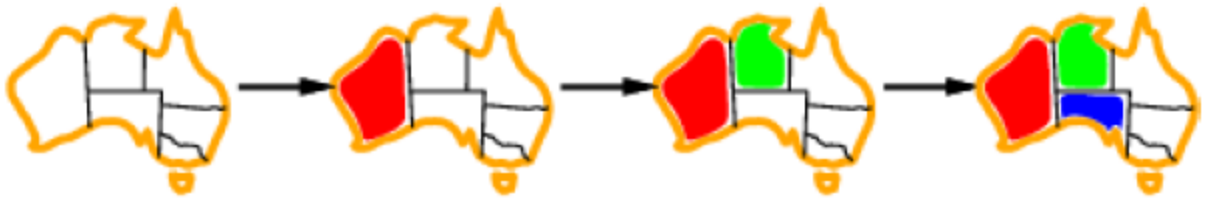
Most constraining variable

- Choose the variable that constrains the most remaining variables
- SA places constrains on 5 variables; WA, NT, Q, V, NSW only on 3, T on 0.
- Purpose is to prune early by reducing number of choices.
- Tie breaker among minimum remaining variables (MRV)



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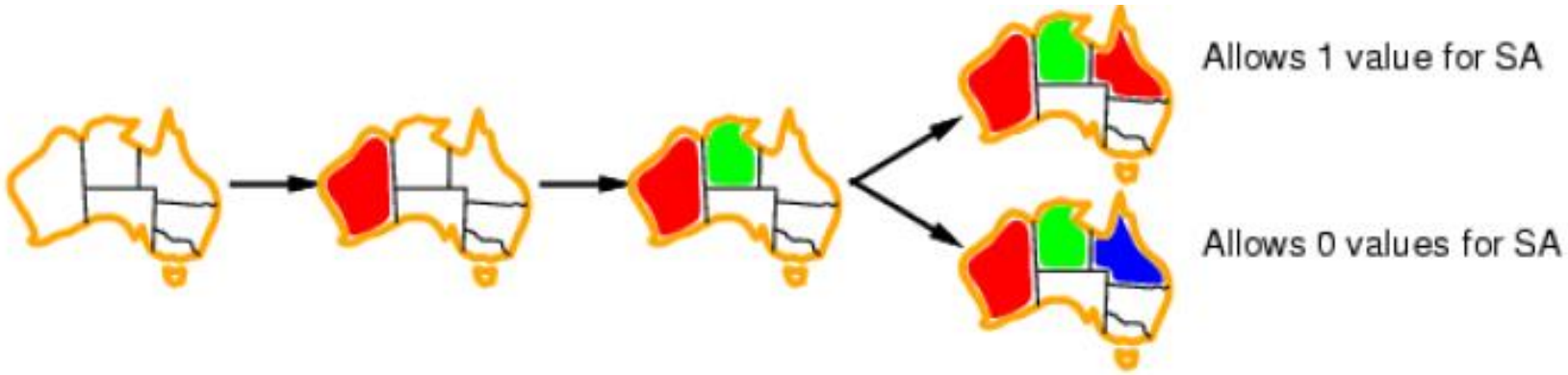
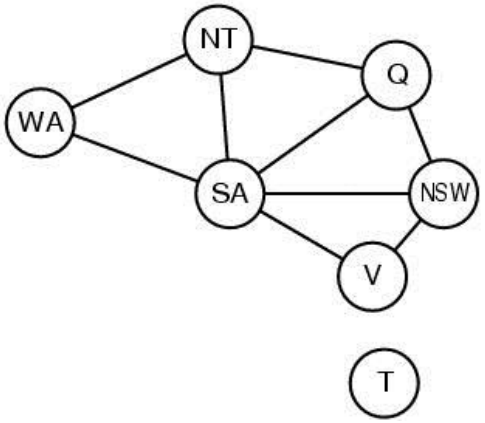
Least Constraining Value

- Heuristic Rule:
- Given a variable, choose the least constraining value



Idea is to explore paths, those with most options, most likely to lead to **a solution.**

Forward checking



- Can we detect inevitable failure early?
  - *And avoid it later?*
- ***Forward checking idea***: keep track of remaining legal values for unassigned variables.
- Terminate search when any variable has *no legal values*.

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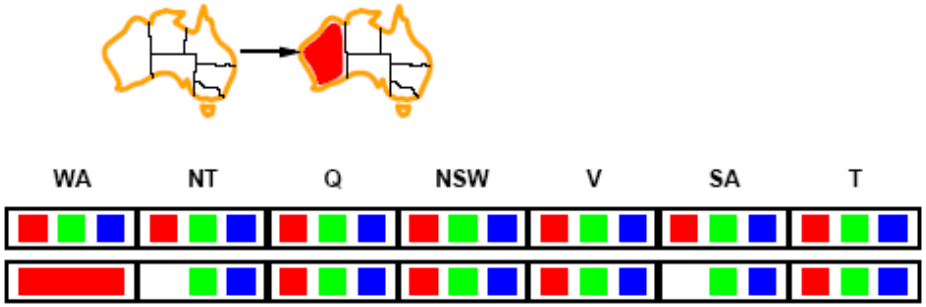
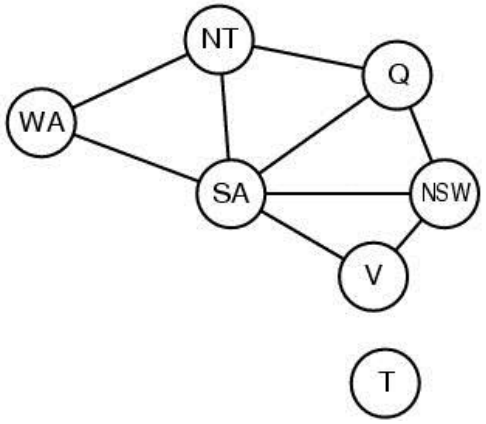
Forward checking





- Assign {*WA=red*}

- Effects on other variables connected by constraints to WA
  - *NT can no longer be red*
  - *SA can no longer be red*

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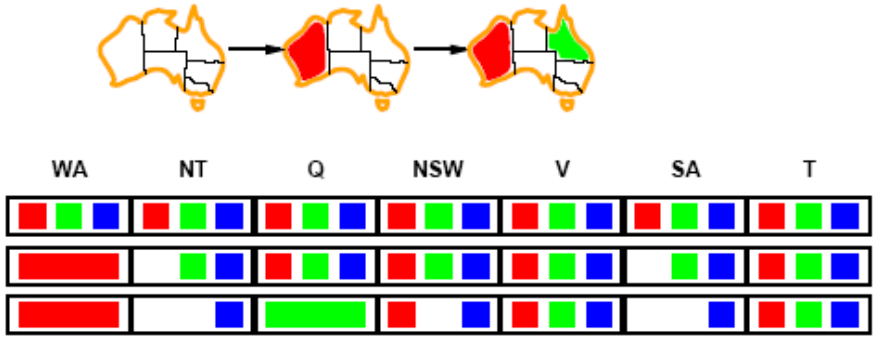
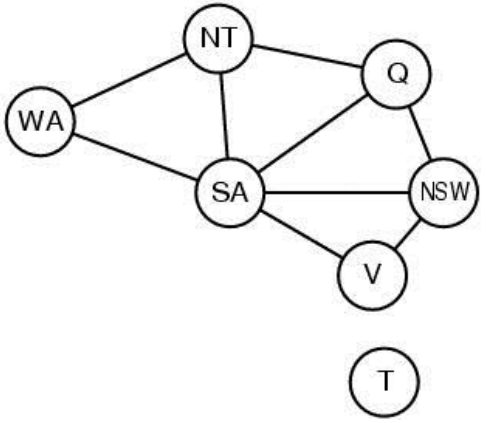
Forward checking





- Assign {Q=*green*}

- Effects on other variables connected by constraints with WA
  - *NT can no longer be green*
  - *NSW can no longer be green*
  - *SA can no longer be green*

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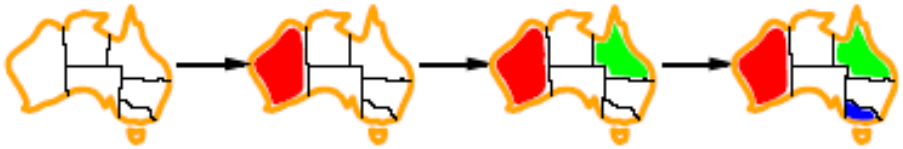
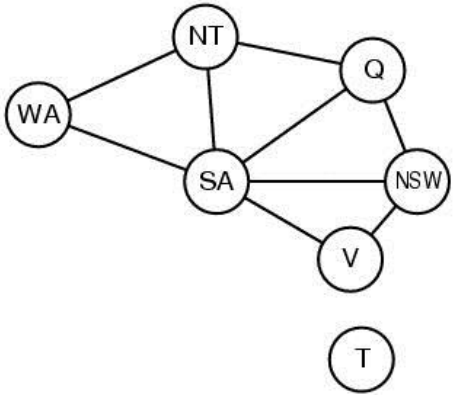
Forward checking



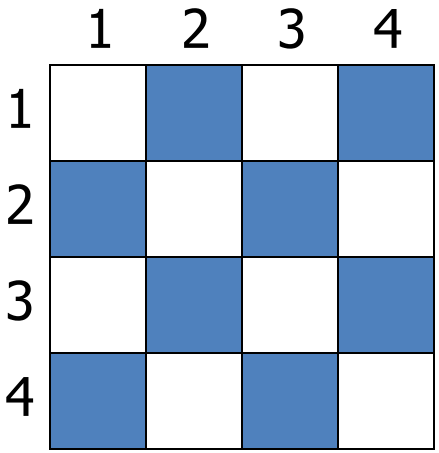


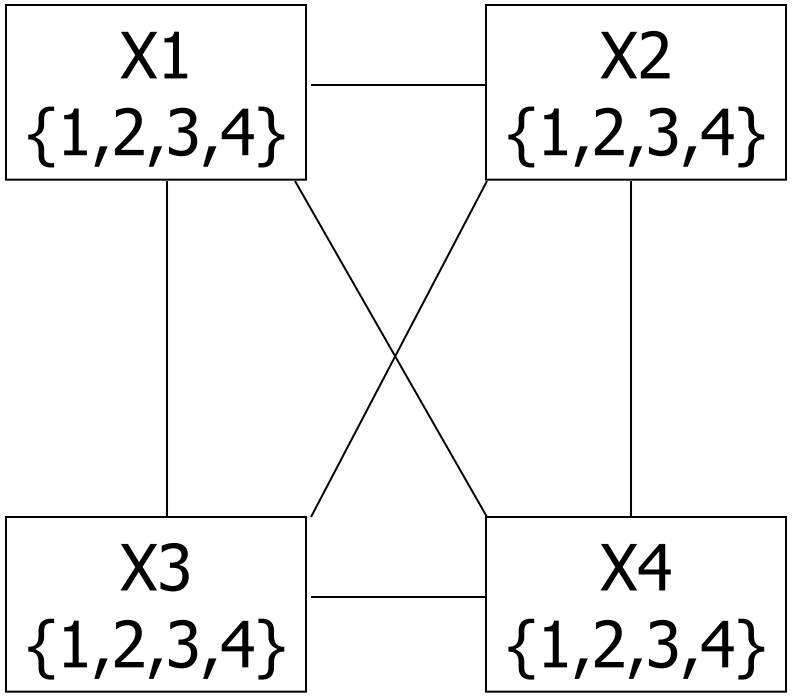
- If *V* is assigned *blue*
- Effects on other variables connected by constraints with WA
  - *NSW can no longer be blue*
  - *SA is* ***empty***

- FC has detected that partial assignment is *inconsistent* with the constraints and backtracking can occur.

Forward checking

Example: 4-Queens Problem





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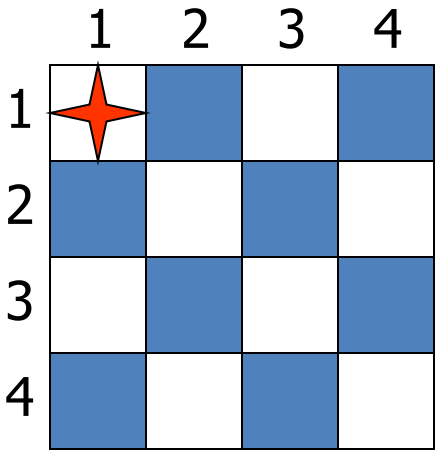
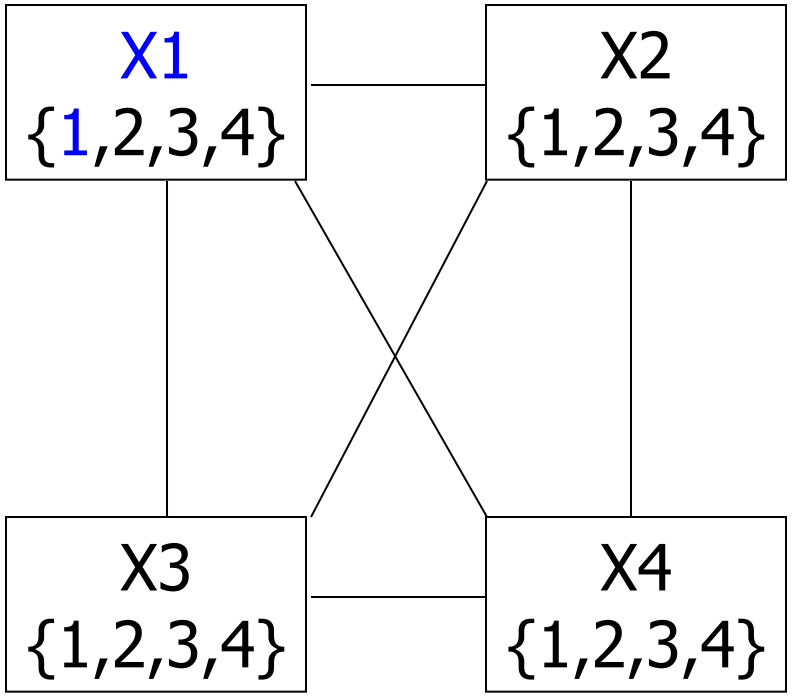
Example: 4-Queens Problem

Forward checking





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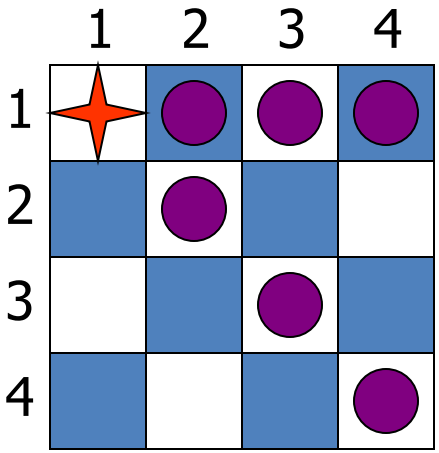

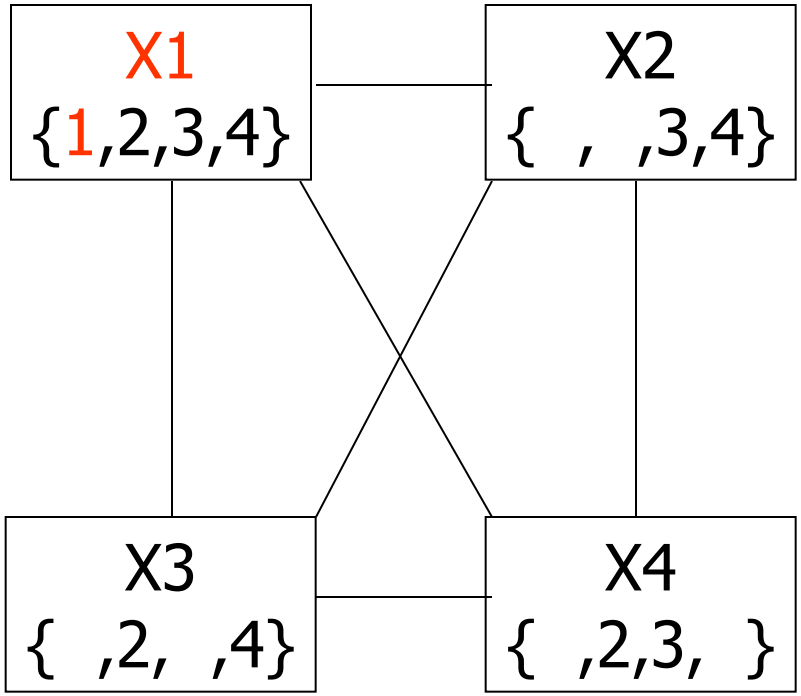

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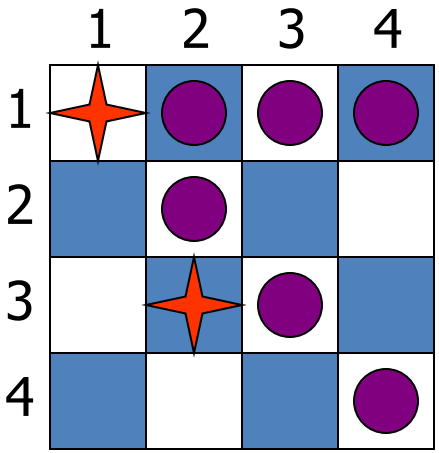

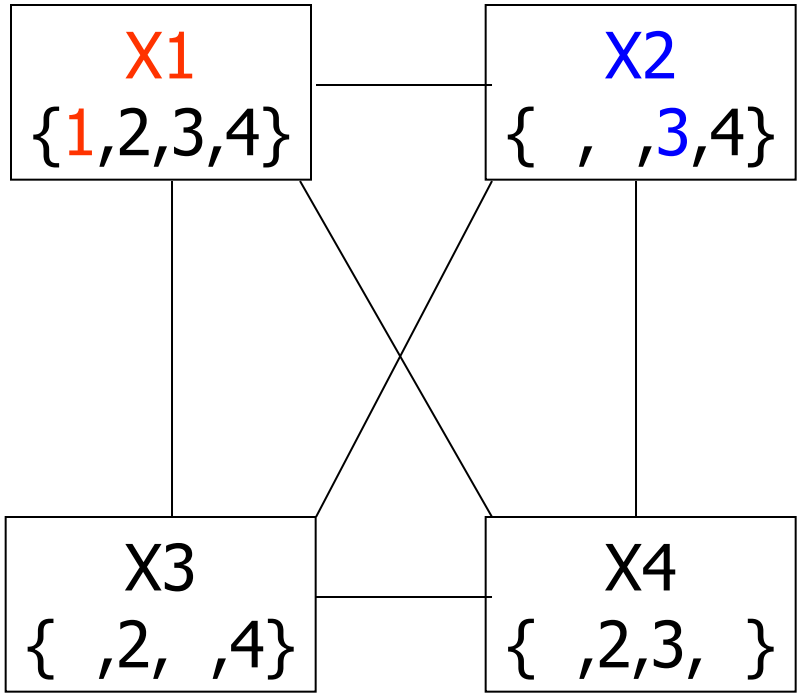

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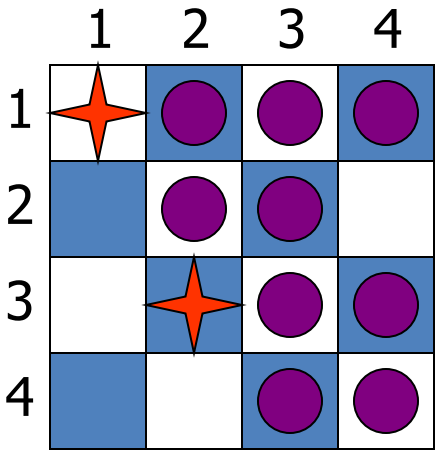

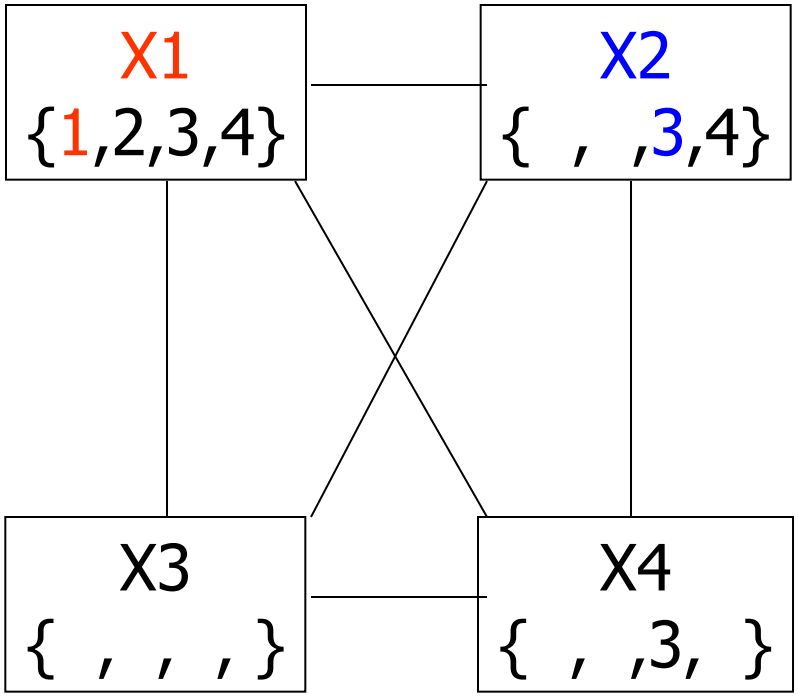

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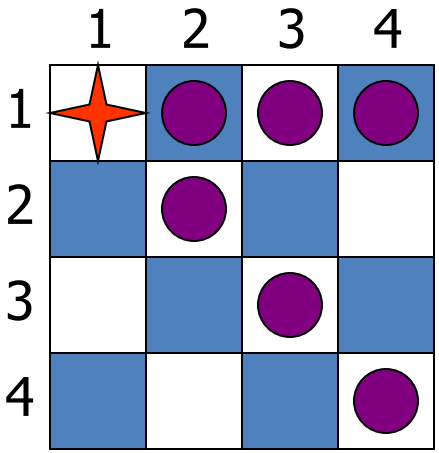

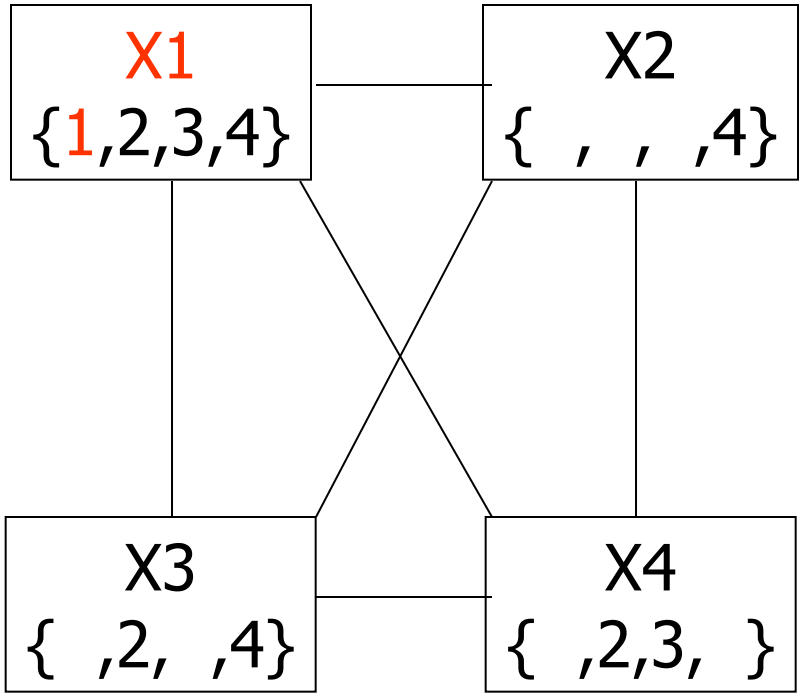

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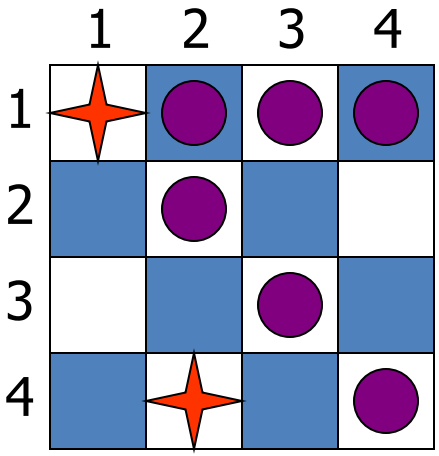

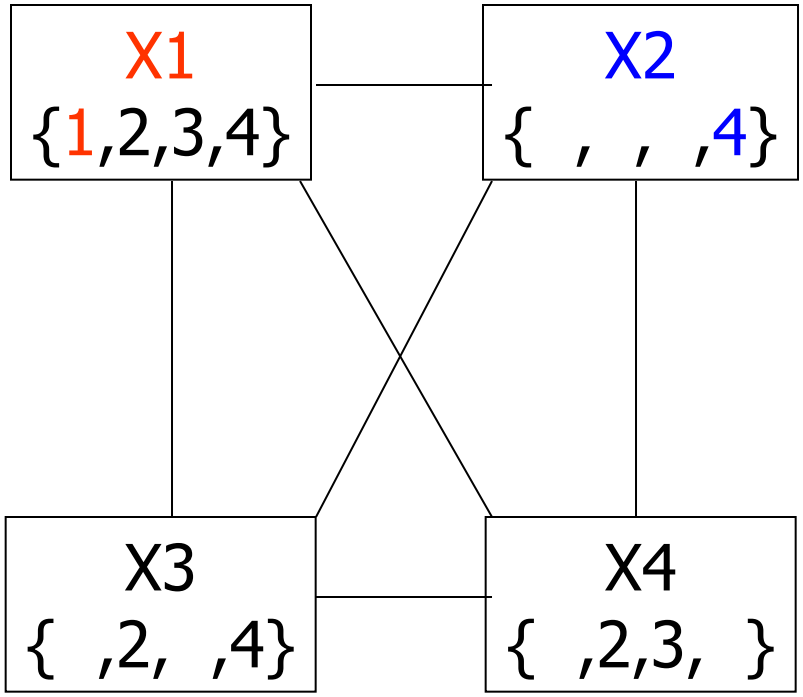

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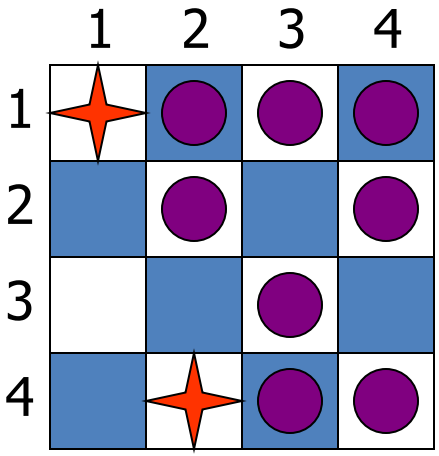

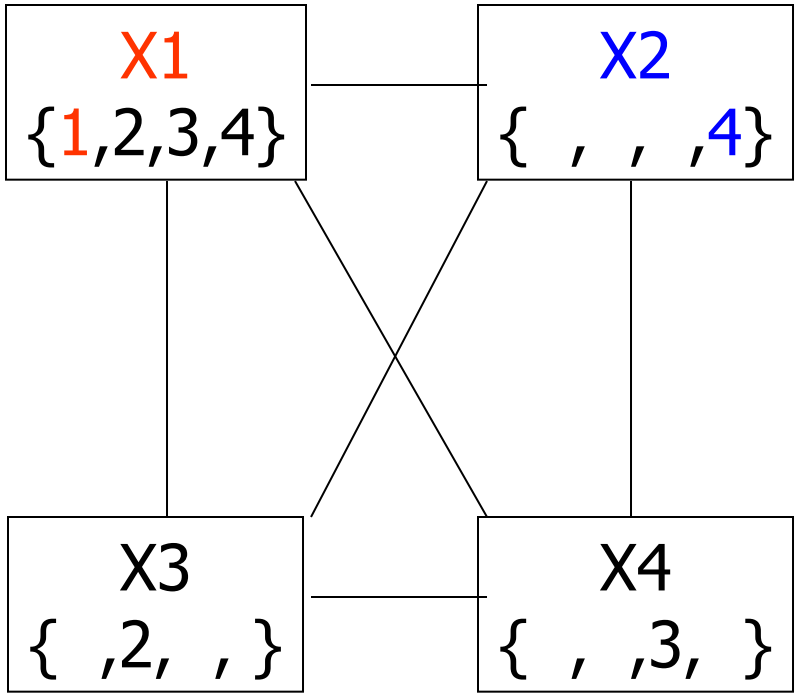

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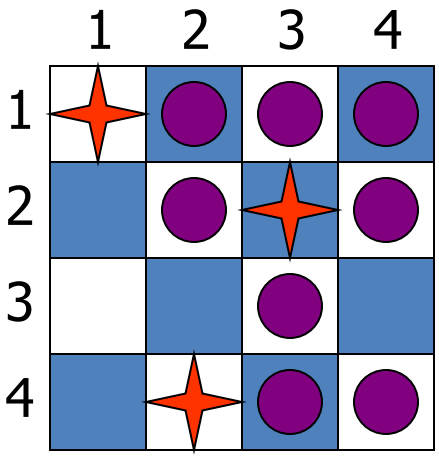

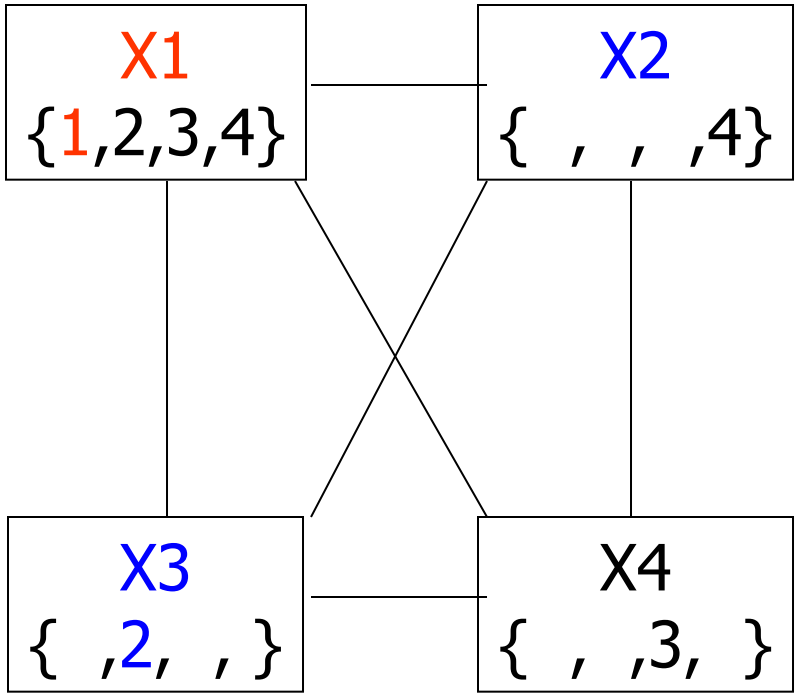

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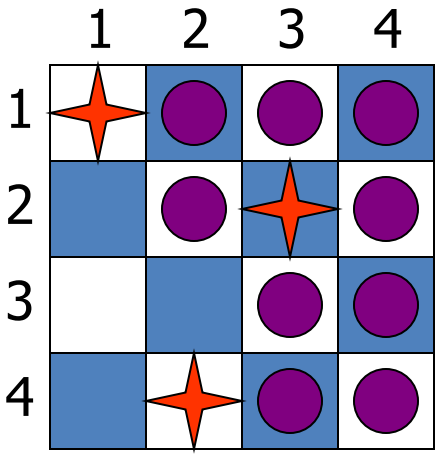

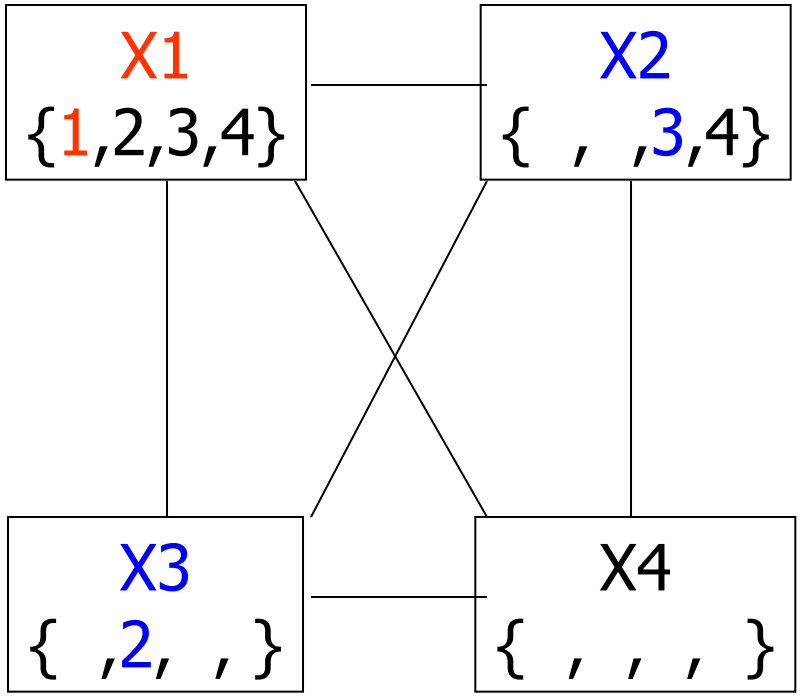

Dead End → Backtrack

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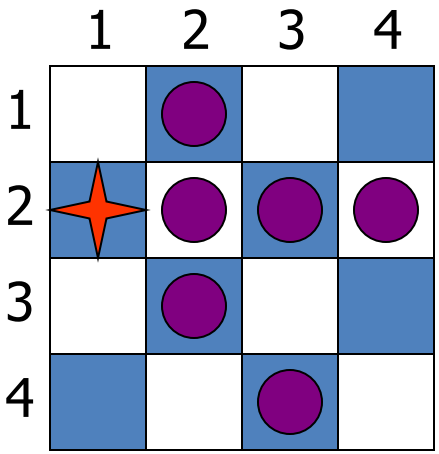
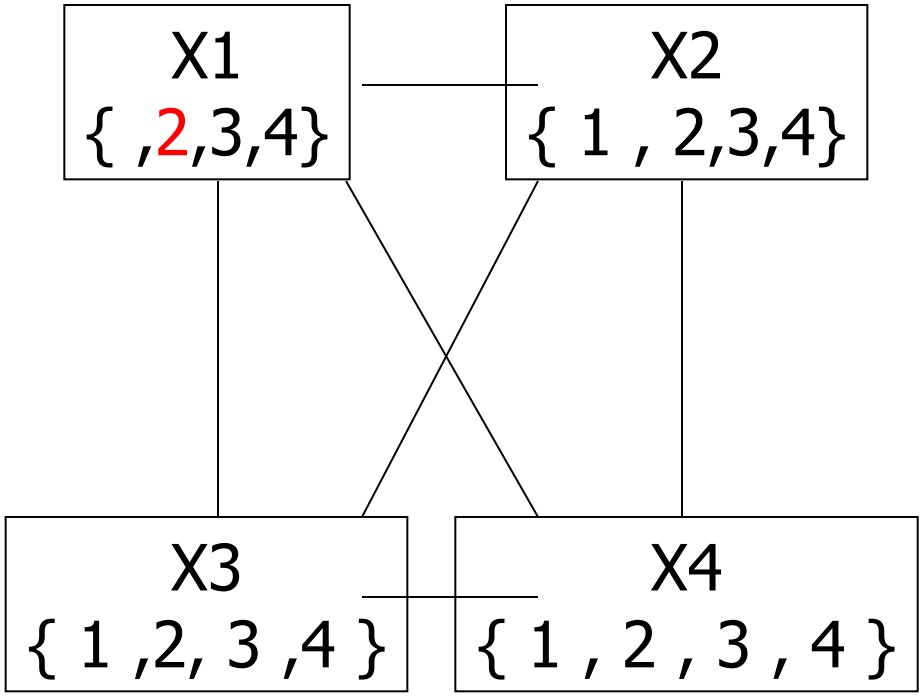
Example: 4-Queens Problem





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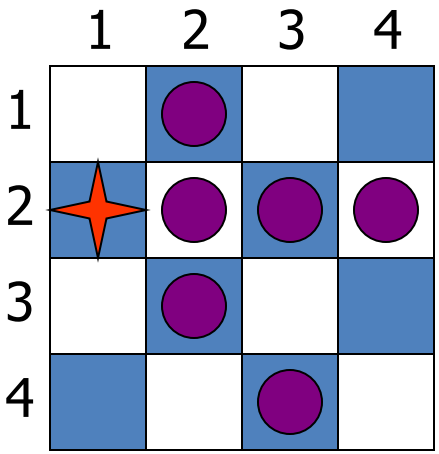
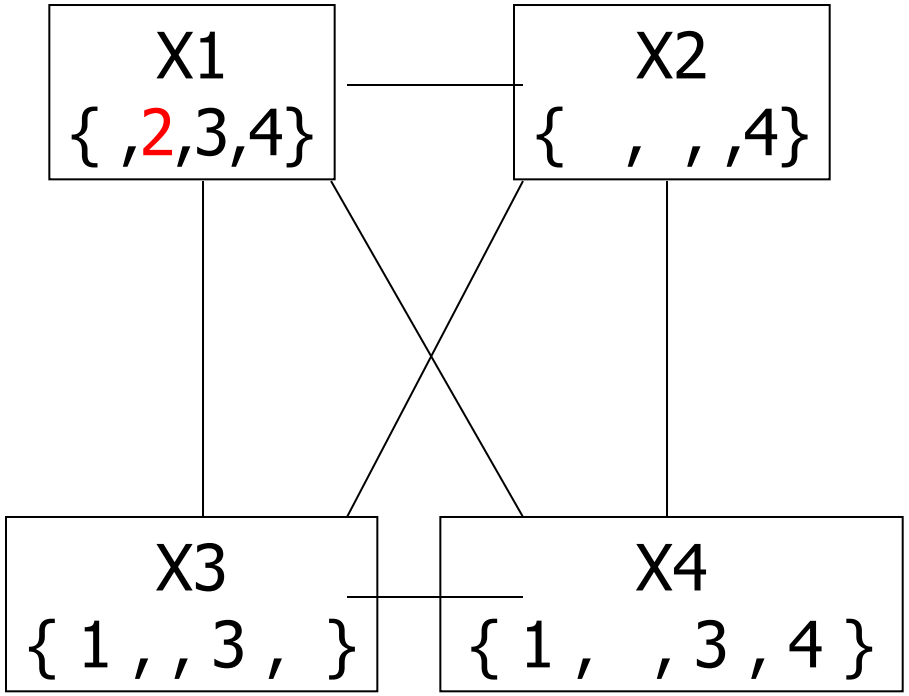
Example: 4-Queens Problem





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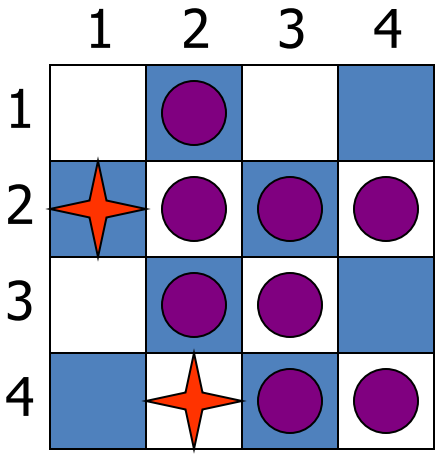
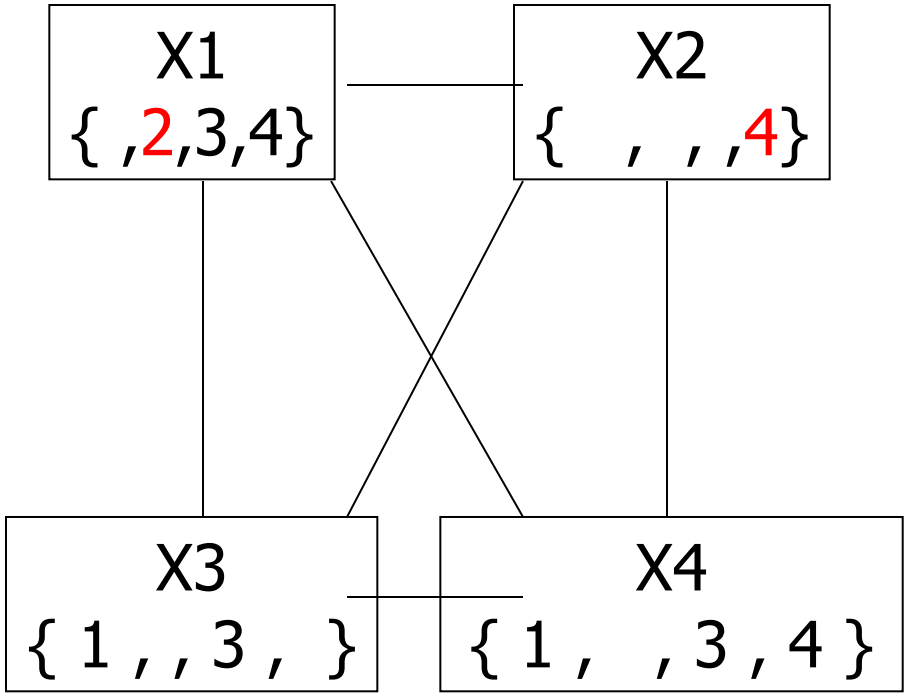
Example: 4-Queens Problem





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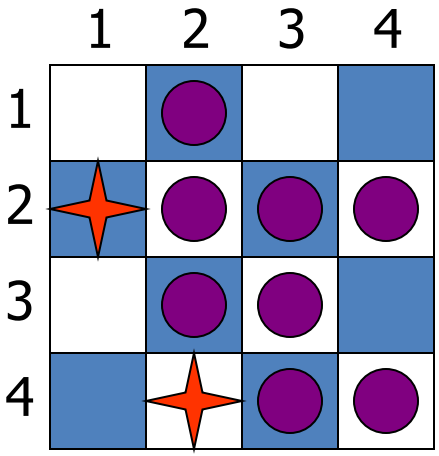
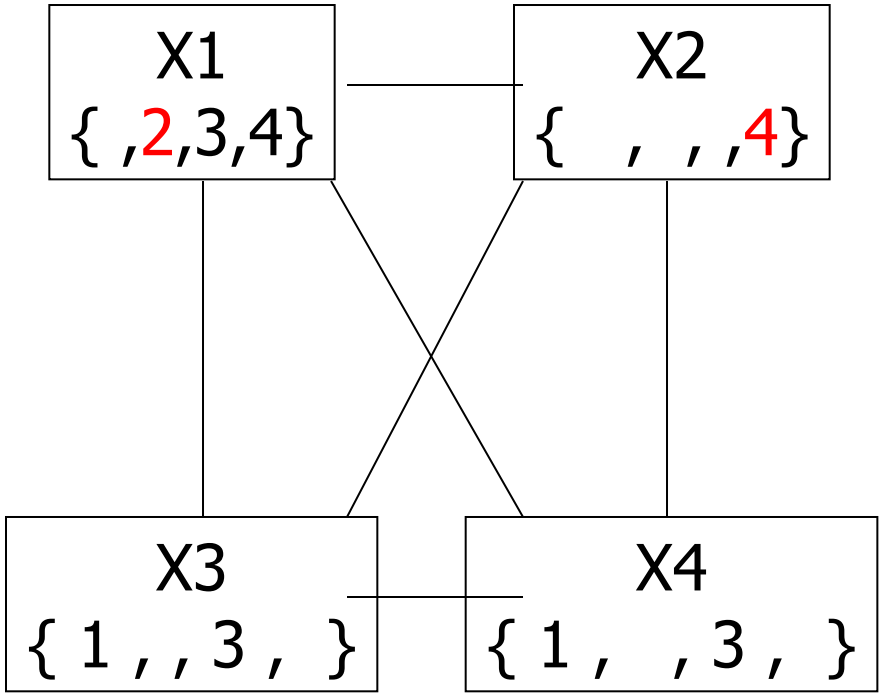
Example: 4-Queens Problem





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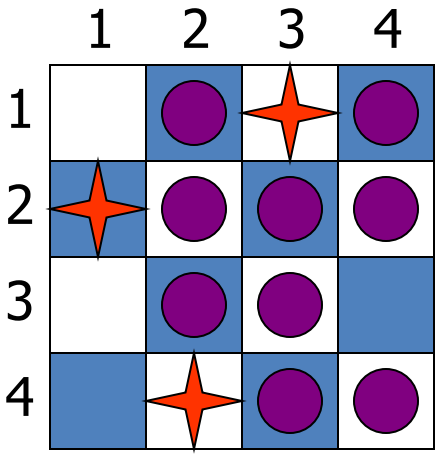
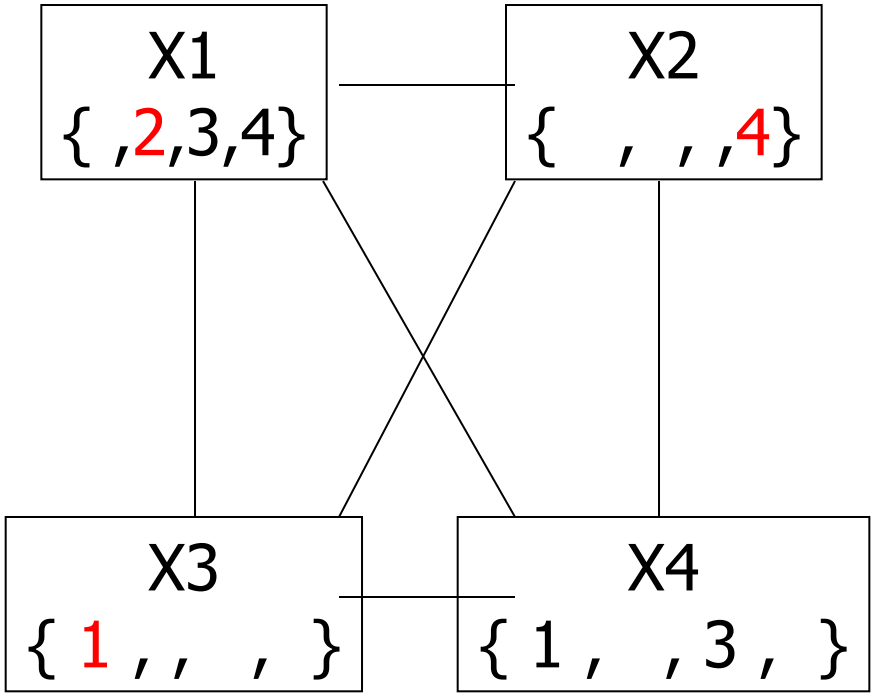
Example: 4-Queens Problem





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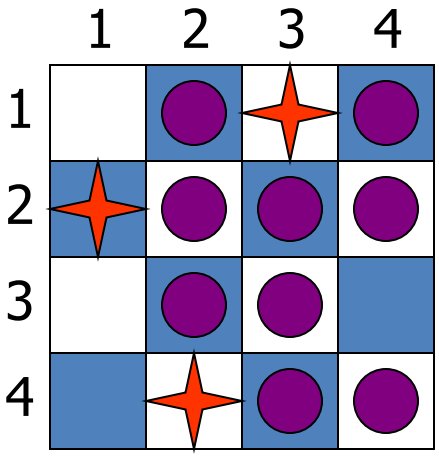
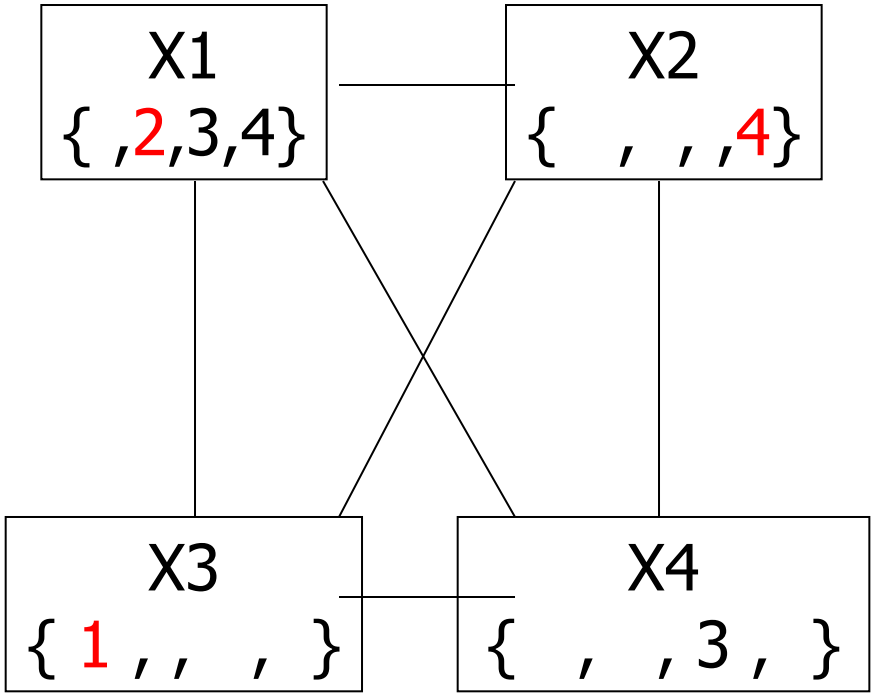
Example: 4-Queens Problem





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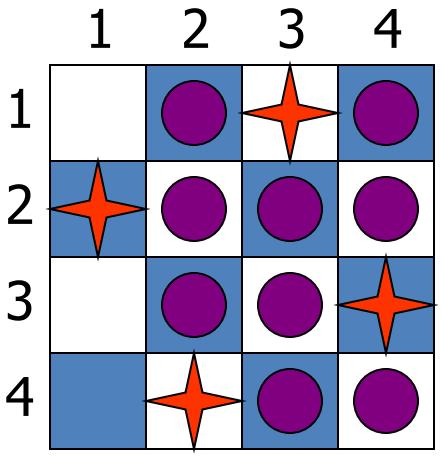
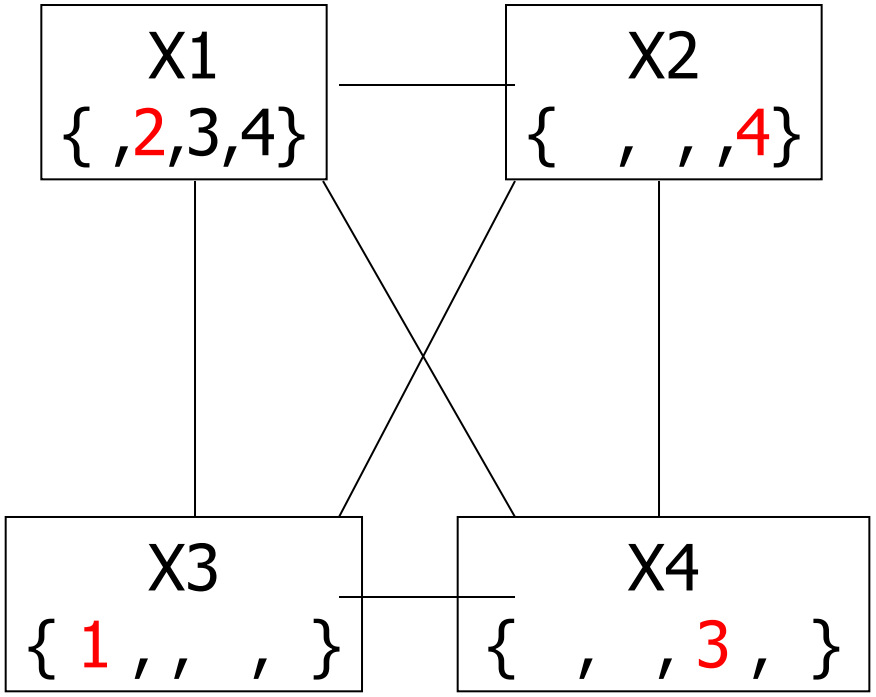
Example: 4-Queens Problem





Solution !!!!

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Consistency Techniques

- removing inconsistent values from variables' domains
- graph representation of the CSP

- **arc consistency (AC)**

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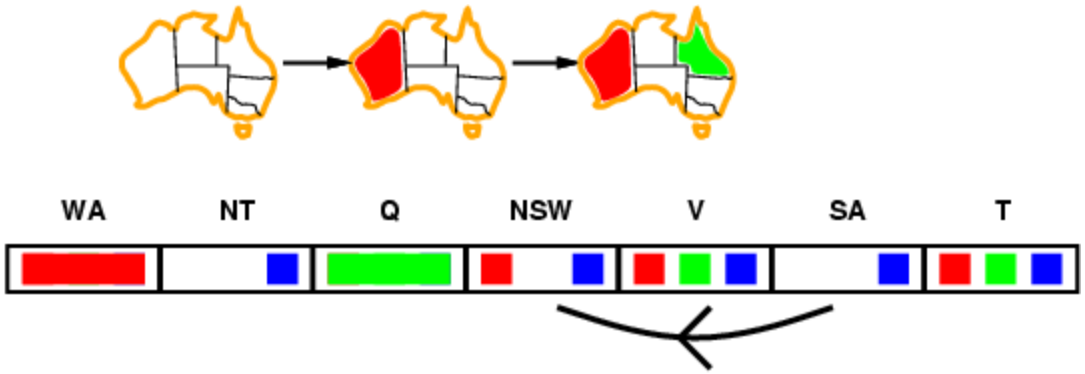
Arc consistency

- Simplest form of propagation makes each arc consistent
- $X \rightarrow Y$ is consistent iff

  for every value x of X there is some allowed y



Arc from SA to NSW is consistent because SA (blue) allows NSW (red)

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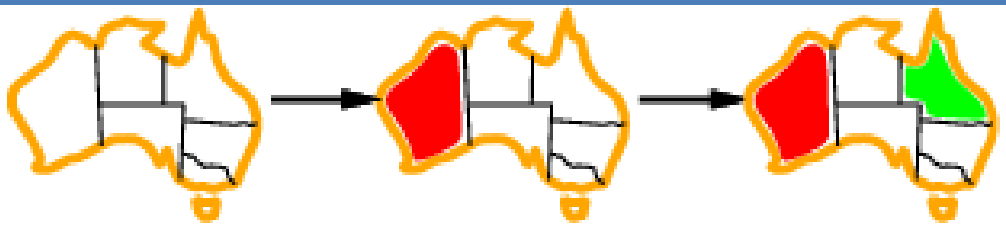
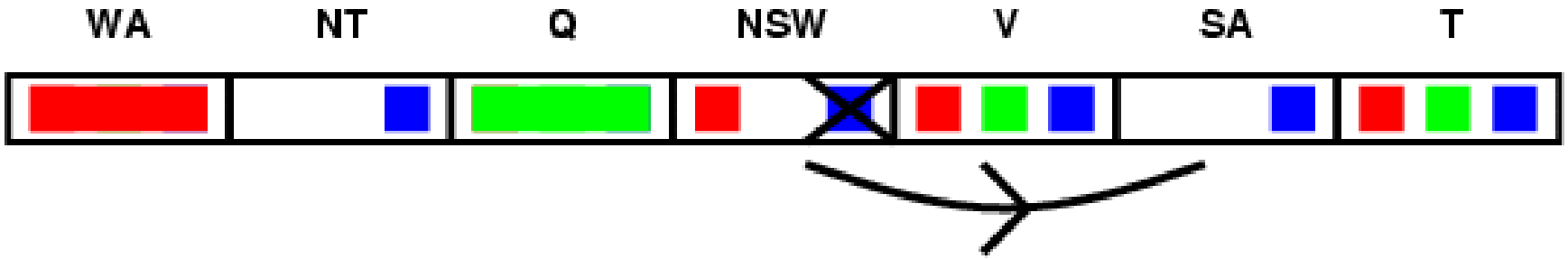
Arc consistency





Arc from NSW to SA is **not consistent** because NSW (blue) does not allow SA (blue)

Can be made consistent by removing blue from NSW domain

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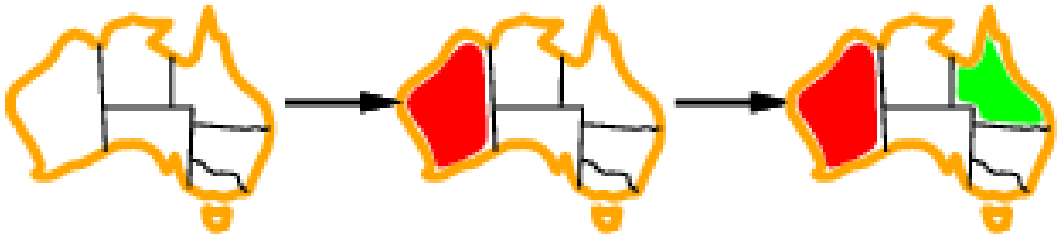
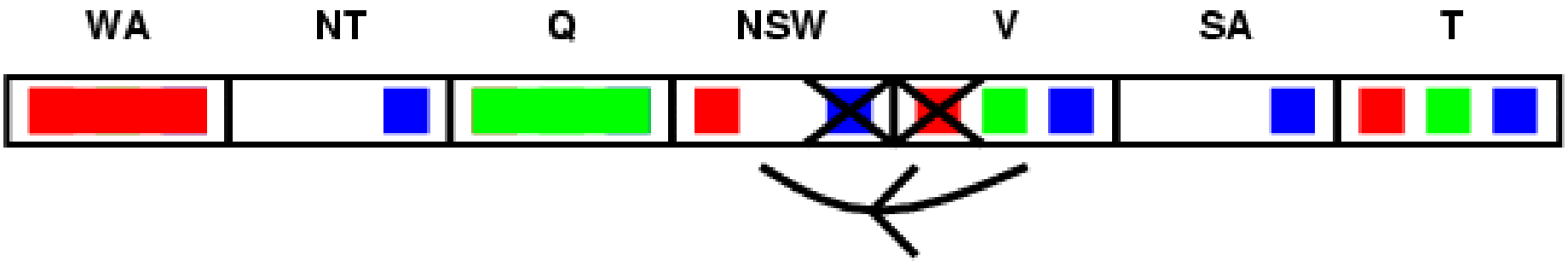
Arc consistency

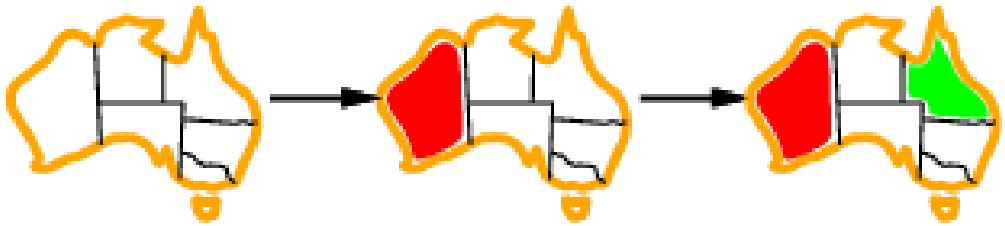


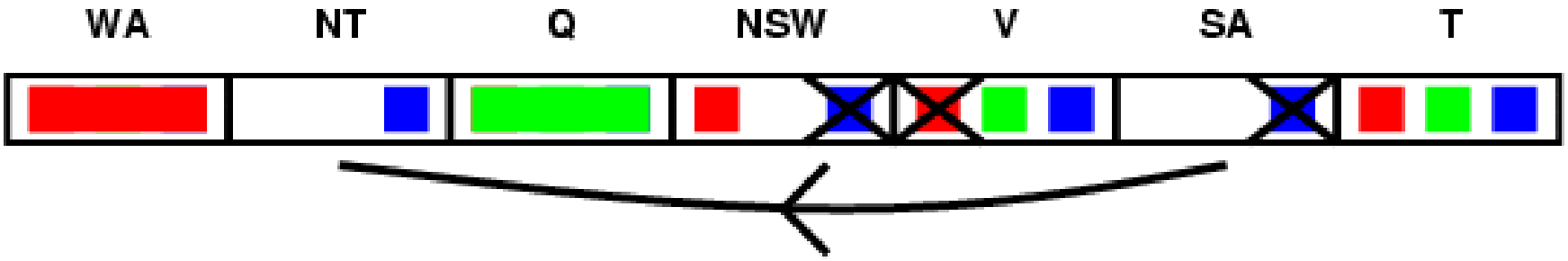


If X loses a value, neighbors of X need to be rechecked.

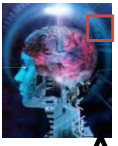
After removing blue from NSW, must check neighbors for consistency leading to removal of red from V.

Arc consistency





- If *X* loses a value, neighbors of *X* need to be rechecked
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment



After removing blue, SA domain is {}, pruning SA completely for the two assignments {NSW=red, Q=green} and forcing backtracking.

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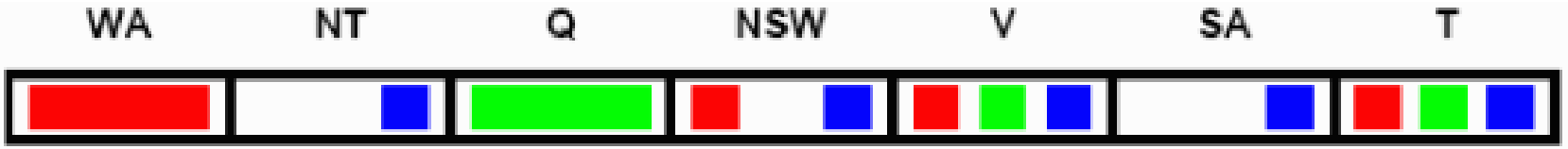

- Is the arc (WA,NSW) consistent?
- Is the arc (NSW,WA) consistent?
- If not, what should be the domain of NSW and WA?
- Is the arc (SA,T) consistent?
- is the arc (T, SA) consistent?

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Thank you

End of Chapter 6

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