Princess Nora University
Faculty of Computer & Information Systems

جامعة الأميرة نورة بنت عبد الرحمن
Princess Nora Bint Abdul Rahman University

ARTIFICIAL INTELLIGENCE

(CS 370D)

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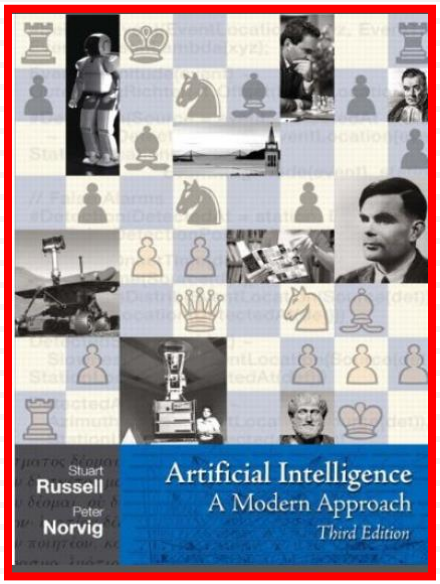

College of Computer and Information Sciences

(CHAPTER-5)
ADVERSARIAL SEARCH

ADVERSARIAL SEARCH

- Optimal decisions
- MinMax algorithm
- α-β pruning
- Imperfect, real-time decisions

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ADVERSARIAL SEARCH

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- Search problems seen so far:
  - ✓ Single agent.
  - ✓ No interference from other agents and no competition.

- Game playing:
  - ✓ Multi-agent environment.
  - ✓ Cooperative games.
  - ✓ Competitive games → **adversarial search**

- **Specifics of adversarial search:**
  - ✓ Sequences of player's decisions we control.
  - ✓ Decisions of other players we do not control.

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Games vs. search problems

- "Unpredictable" opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

Game as Search Problem
MinMax Algorithm

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Game setup

- **Consider a game with Two players: (Max)** and **(Min)**
- **Max** moves first and they take turns until the game is over. Winner gets award, loser gets penalty.
- **Games as search:**
  - **Initial state**: e.g. board configuration of chess
  - **Successor function**: list of (move, state) pairs specifying legal moves.
  - **Goal test**: Is the game finished?
  - **Utility function**: Gives numerical value of terminal states. E.g. win (+1), lose (-1) and draw (0) in tic-tac-toe

Max uses **search tree** to determine next move.

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Example of an ADVERSARIAL two player Game

Tic-Tac-Toe (TTT)

- MAX has 9 possible first moves, etc.
- Utility value is always from the point of view of MAX.
- High values good for MAX and bad for MIN.

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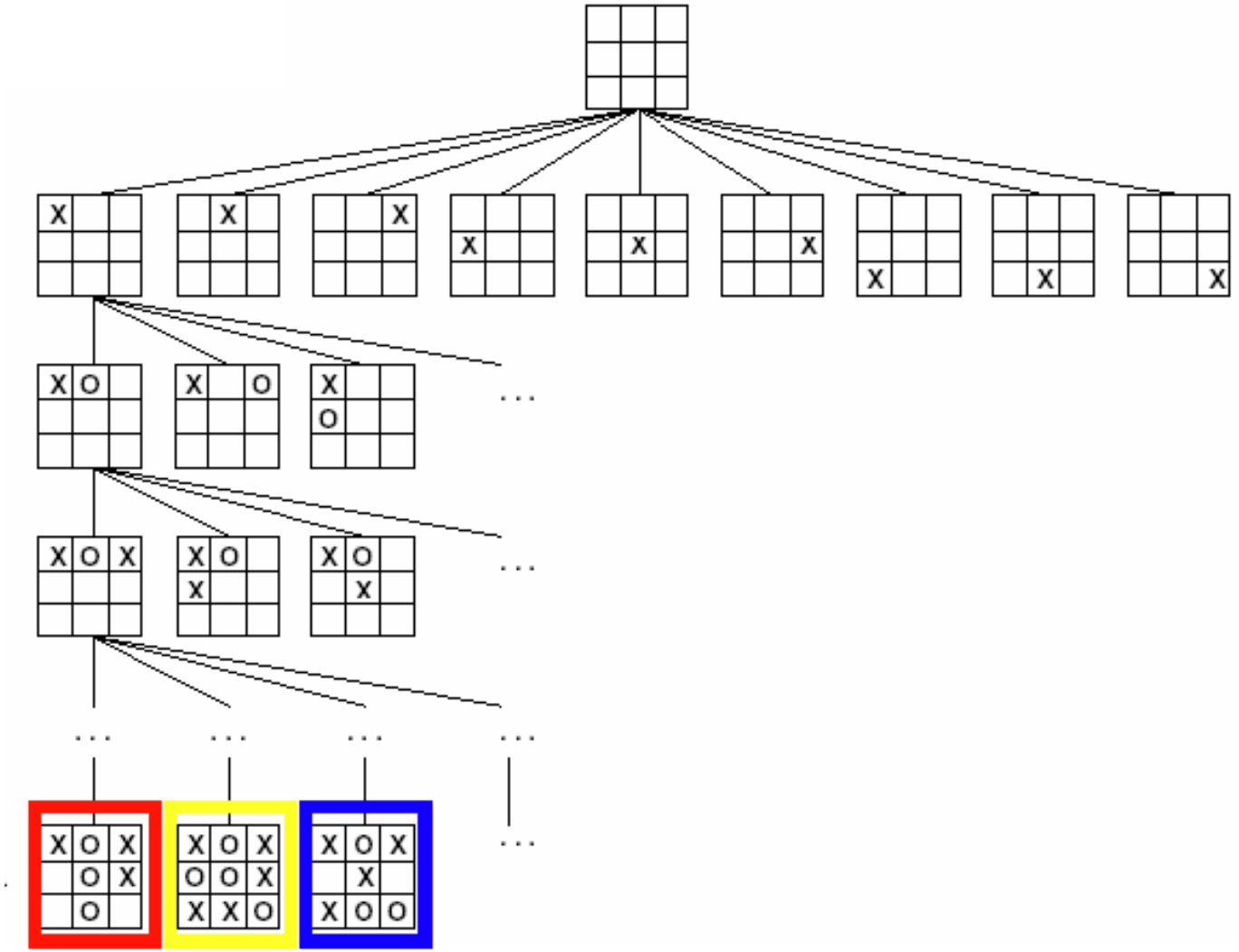
Example of an adversarial 2 person game: Tic-tac-toe



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How to Play a Game by Searching

- **General Scheme**
  - Consider all legal moves, each of which will lead to some new state of the environment ('board position')
  - **Evaluate** each possible resulting board position
  - Pick the move which leads to the best board position.
  - Wait for your opponent's move, then **repeat**.
- **Key problems**
  - Representing the 'board'
  - Representing legal next boards
  - Evaluating positions
  - Looking ahead

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Game Trees

- Represent the problem space for a game by a tree
  - **Nodes** represent 'board positions' (state)
  - **edges** represent legal moves.

- **Root node** is the position in which a decision must be made.

- **Evaluation function *f*** assigns real-number scores to `board positions.'

- **Terminal nodes (leaf)** represent ways the game could end, labeled with the desirability of that ending (e.g. win/lose/draw or a numerical score)

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MAX & MIN Nodes

- When I move, I attempt to **MAXimize** my performance.
- When my opponent moves, he attempts to **MINimize** my performance.

TO REPRESENT THIS:

- If we move first, label the root MAX; if our opponent does, label it MIN.
- Alternate labels for each successive tree level.
  - if the root (level 0) is our turn (MAX), all even levels will represent turns for us (MAX), and all odd ones turns for our opponent (MIN).

Evaluation functions

- Evaluations how good a 'board position' is
  - Based on static features of that board alone
- Zero-sum assumption lets us use one function to describe goodness for both players.
  - **f(n)>0** if we are winning in position n
  - **f(n)=0** if position n is tied
  - **f(n)<0** if our opponent is winning in position n
- Build using expert knowledge (Heuristic),
  - Tic-tac-toe: f(n)=(# of 3 lengths possible for me) - (# 3 lengths possible for you)

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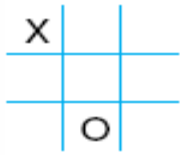
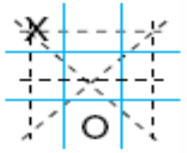
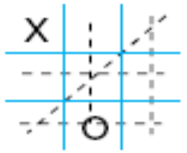
Heuristic measuring for adversarial tic-tac-toe



X has 6 possible win paths:



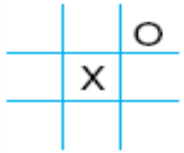
O has 5 possible wins:



E(n) = 6 – 5 = 1

X has 4 possible win paths;
O has 6 possible wins

E(n) = 4 – 6 = –2



X has 5 possible win paths;
O has 4 possible wins

E(n) = 5 – 4 = 1

Heuristic is E(n) = M(n) – O(n)
where M(n) is the total of My possible winning lines
O(n) is total of Opponent's possible winning lines
E(n) is the total Evaluation for state n

Maximize E(n)

E(n) = 0 when my opponent and I have equal number of possibilities.

MinMax Algorithm

- Main idea: choose move to position with highest minimax value. = best achievable payoff against best play.
- E.g., 2-ply game:

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MinMax Algorithm

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

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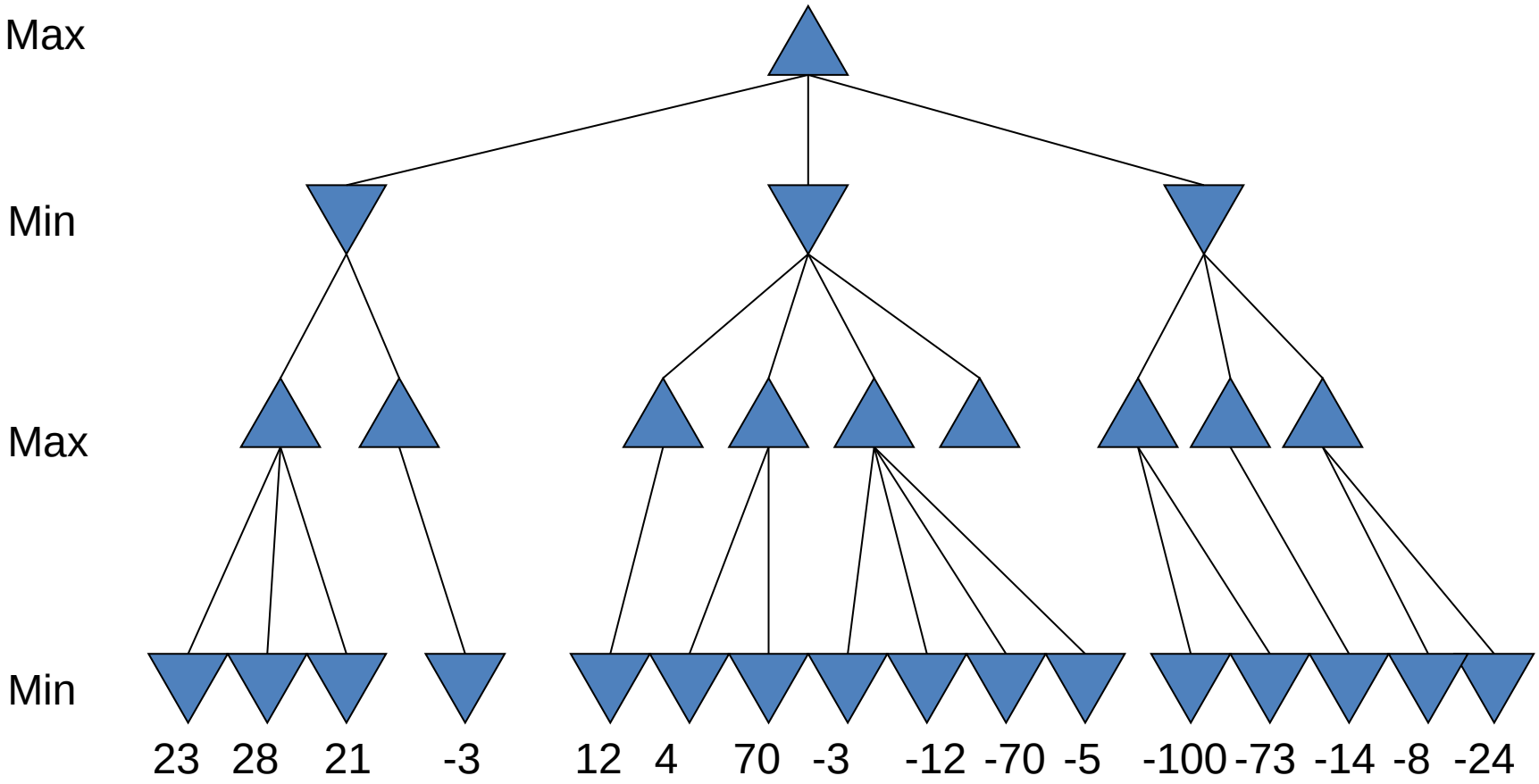
Minimax tree



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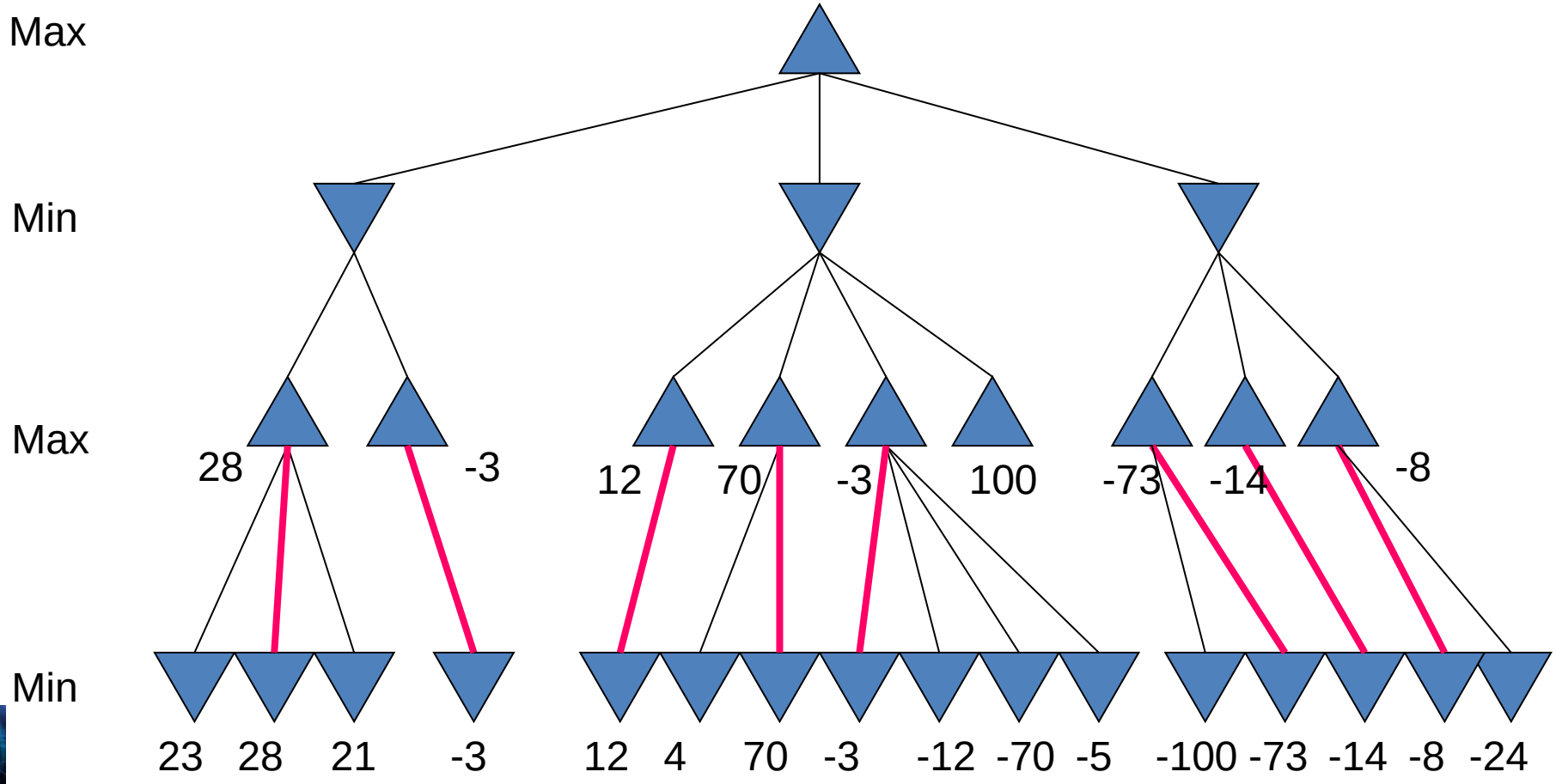
Minimax tree



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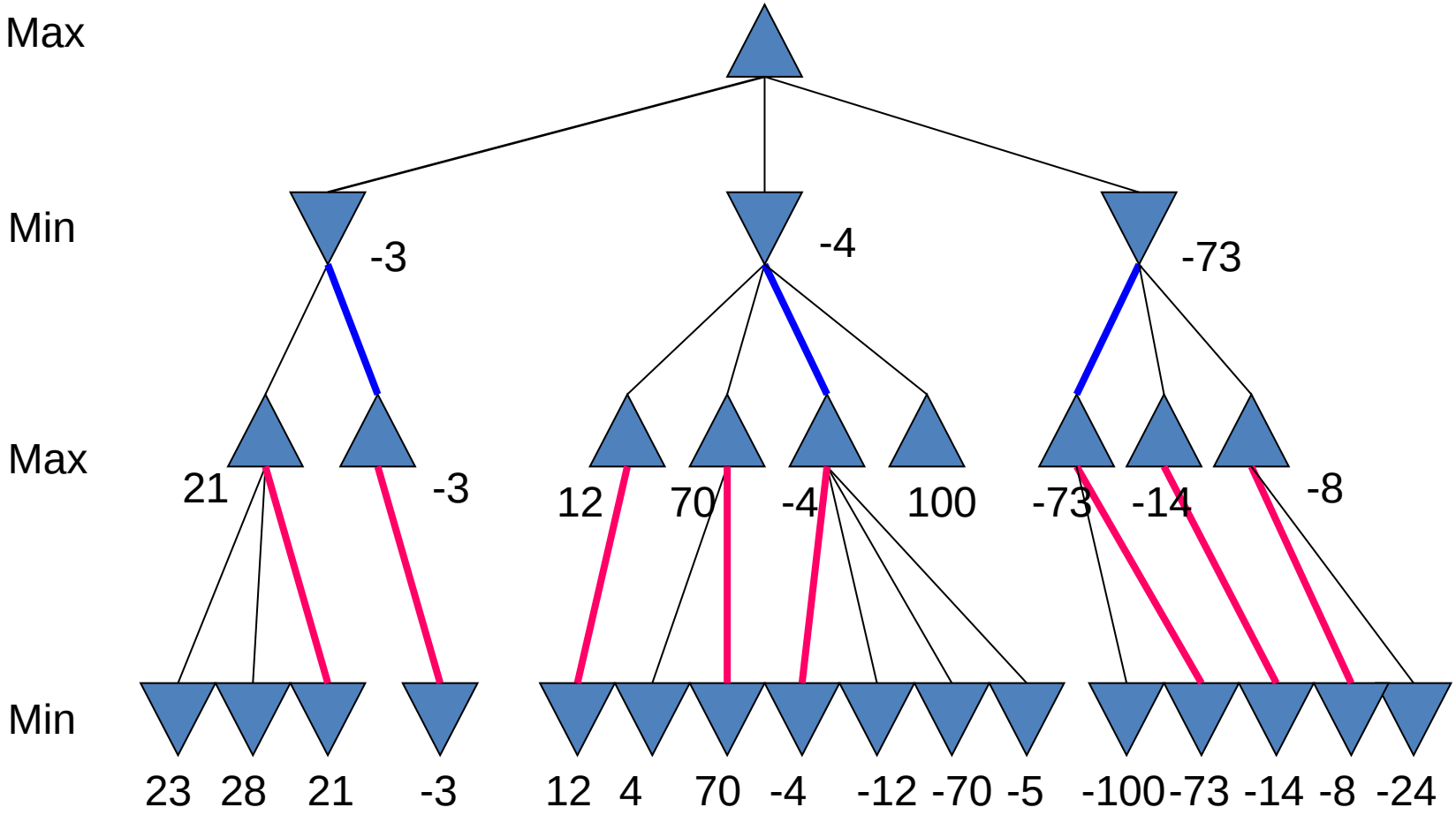
Minimax tree



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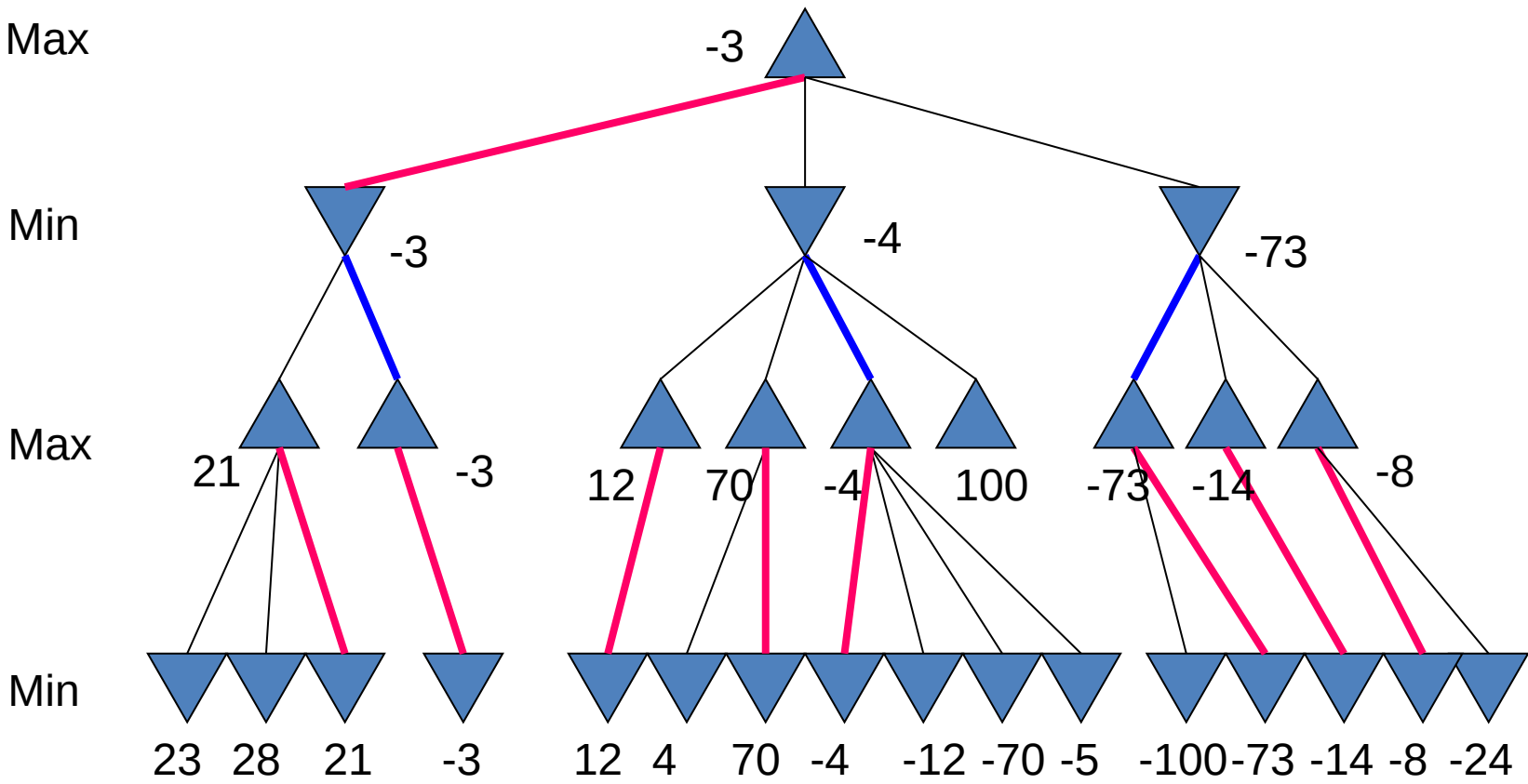
Minimax tree



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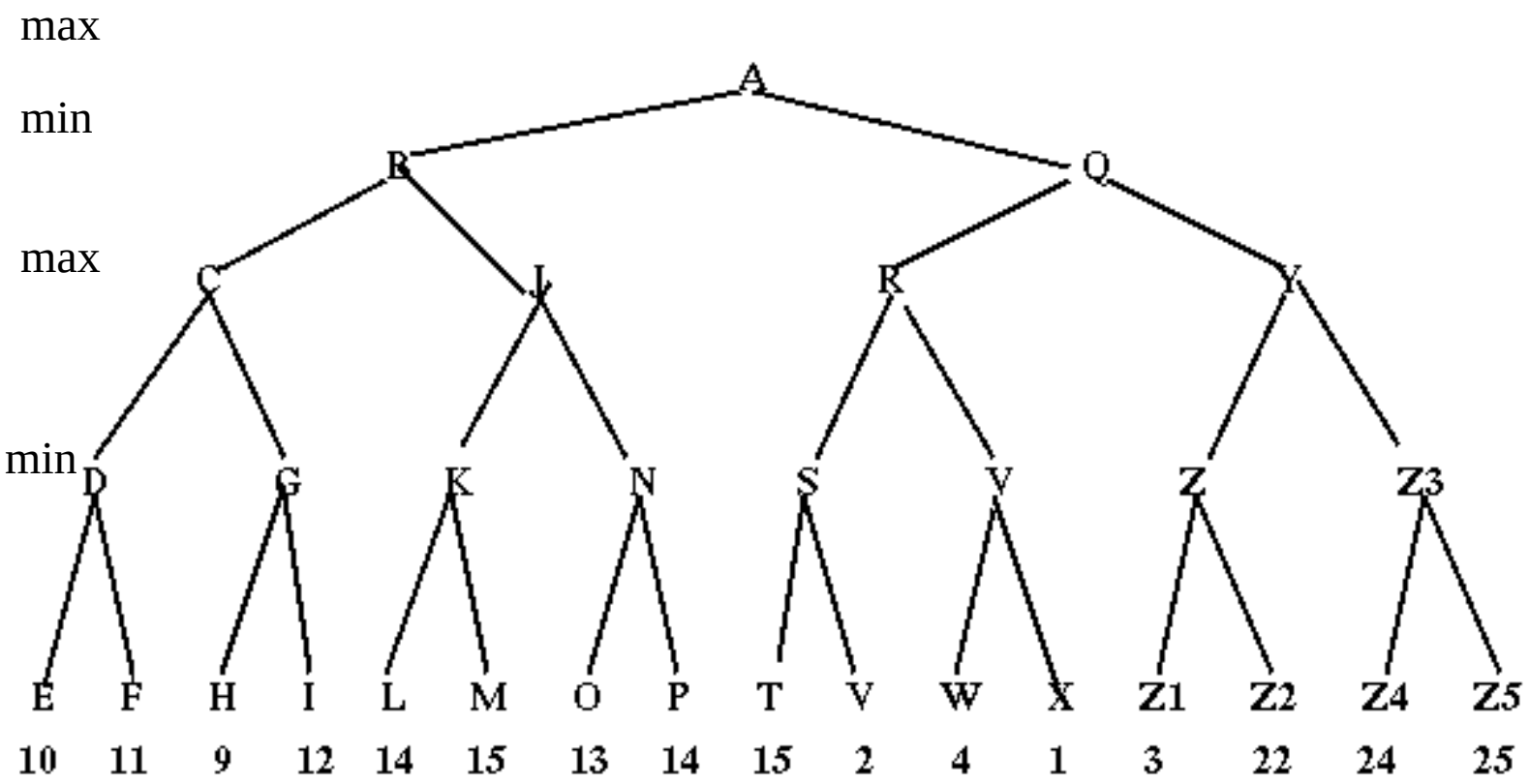
Minimax



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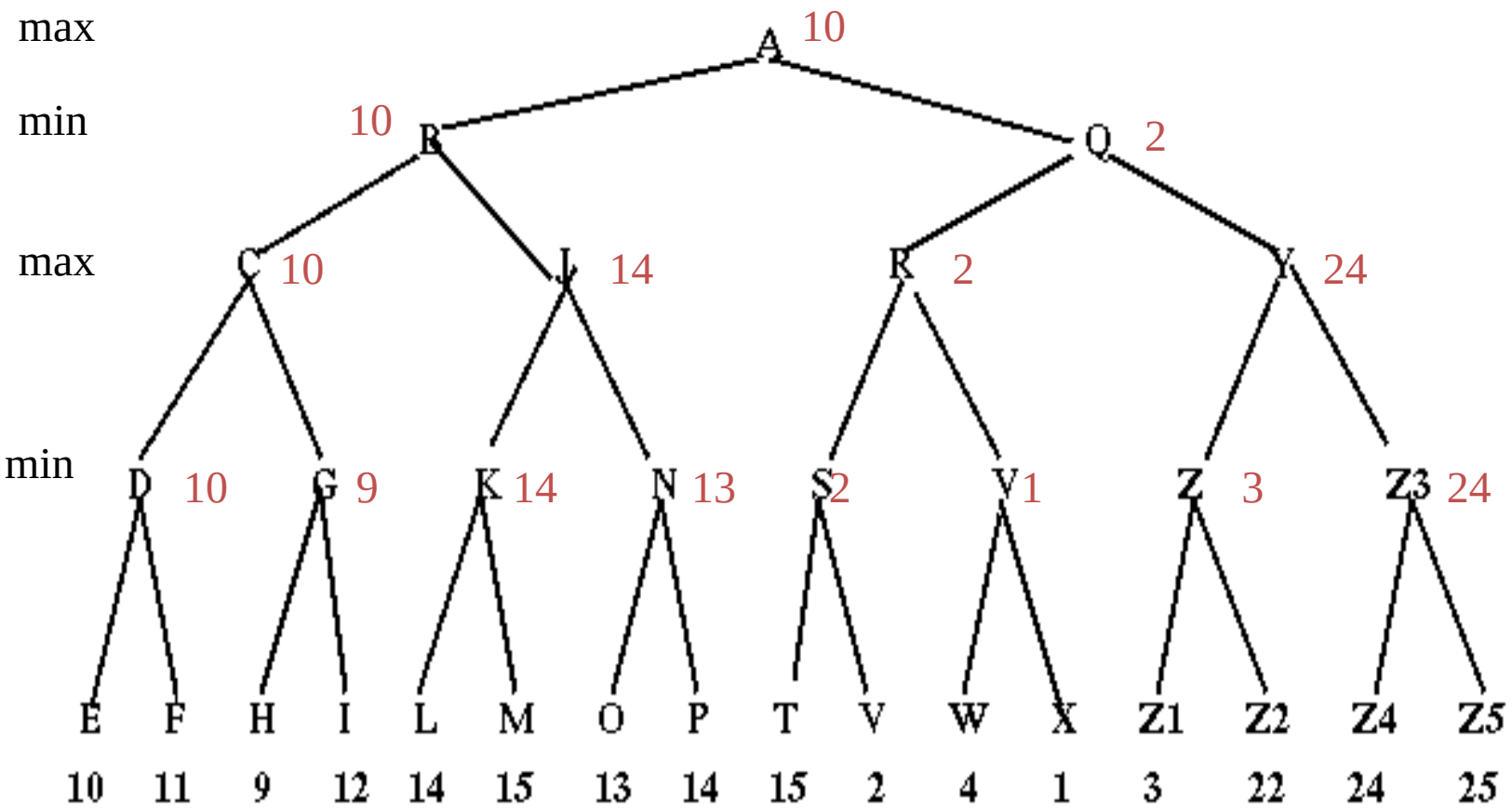
Minimax



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MinMax Analysis

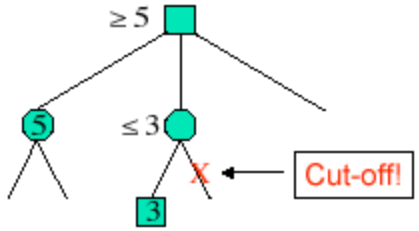
- Time Complexity: $O(b^d)$
- Space Complexity: $O(b*d)$
- Optimality: Yes

Problem: Game → *Resources Limited!*

- Time to make an action is limited

- Can we do better ? Yes !
- How ? Cutting useless branches !

Some nodes in the search can be *proven* to be irrelevent to the outcome of the search



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α Cuts

- If the current max value is greater than the successor's min value, don't explore that min subtree any more

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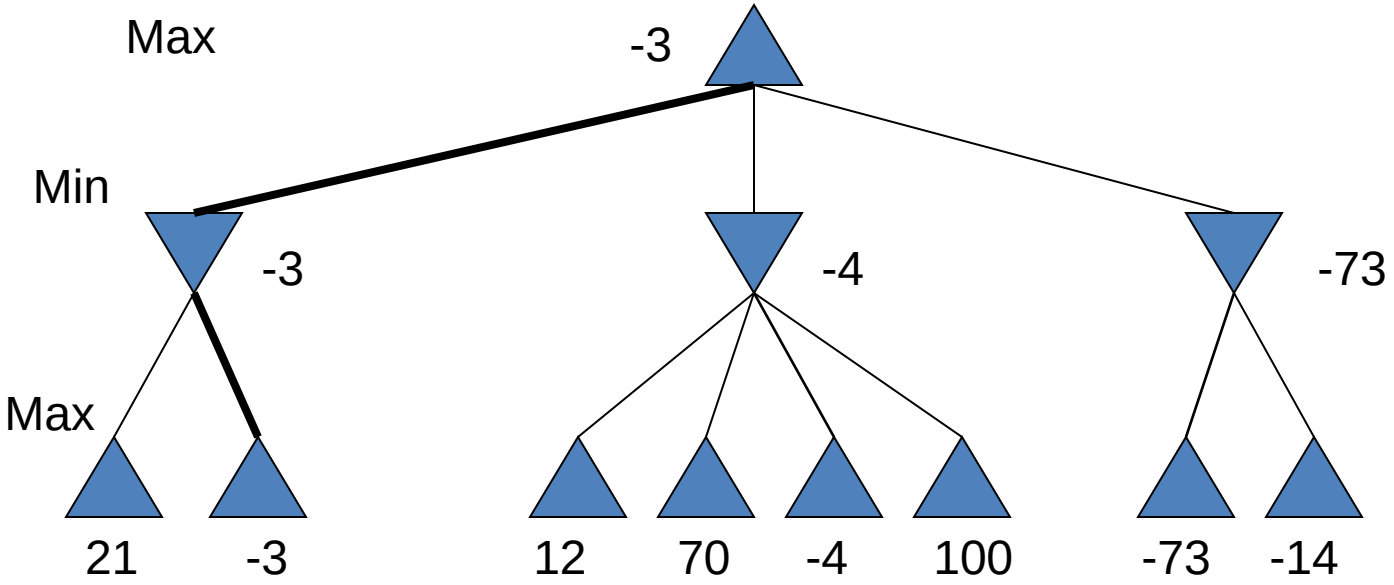
α Cut example



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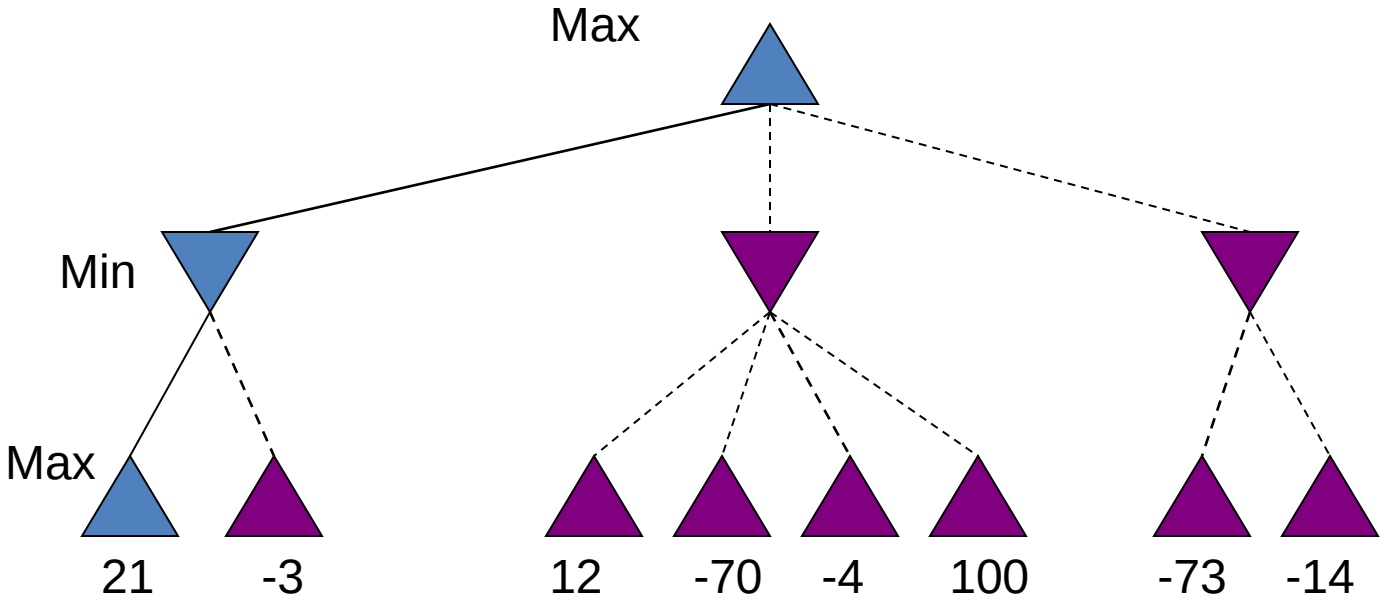
α Cut example



- Depth first search along path 1

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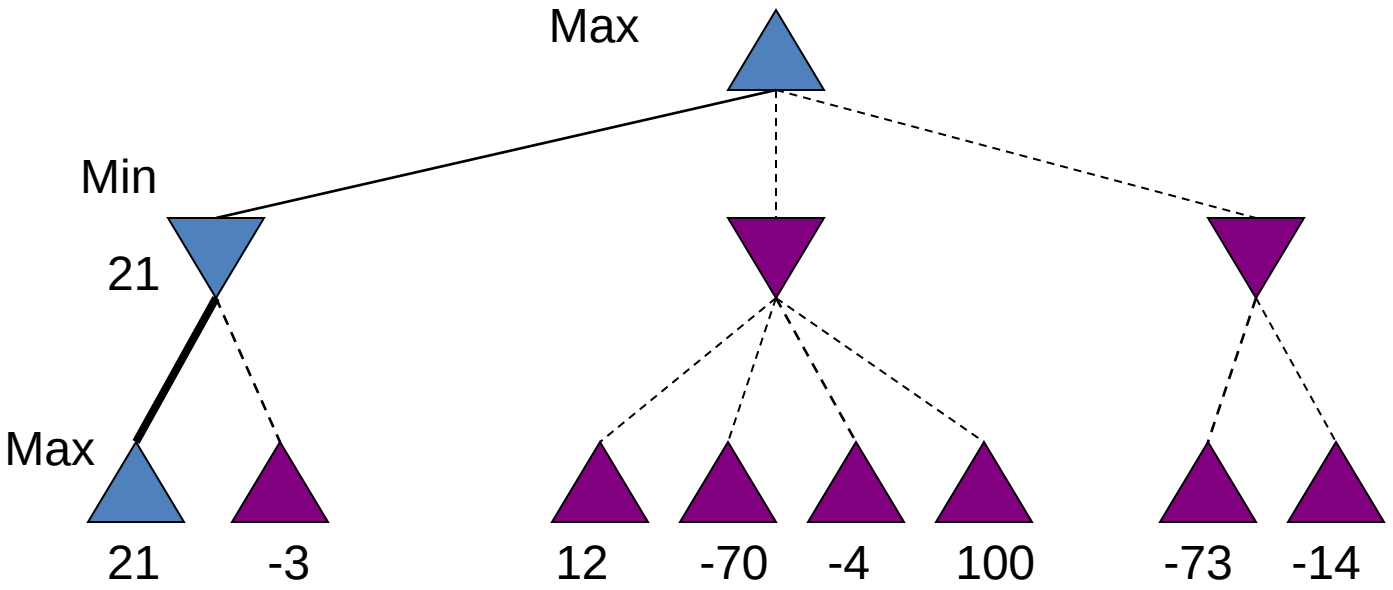
α Cut example



- 21 is minimum so far (second level)
- Can't evaluate yet at top level

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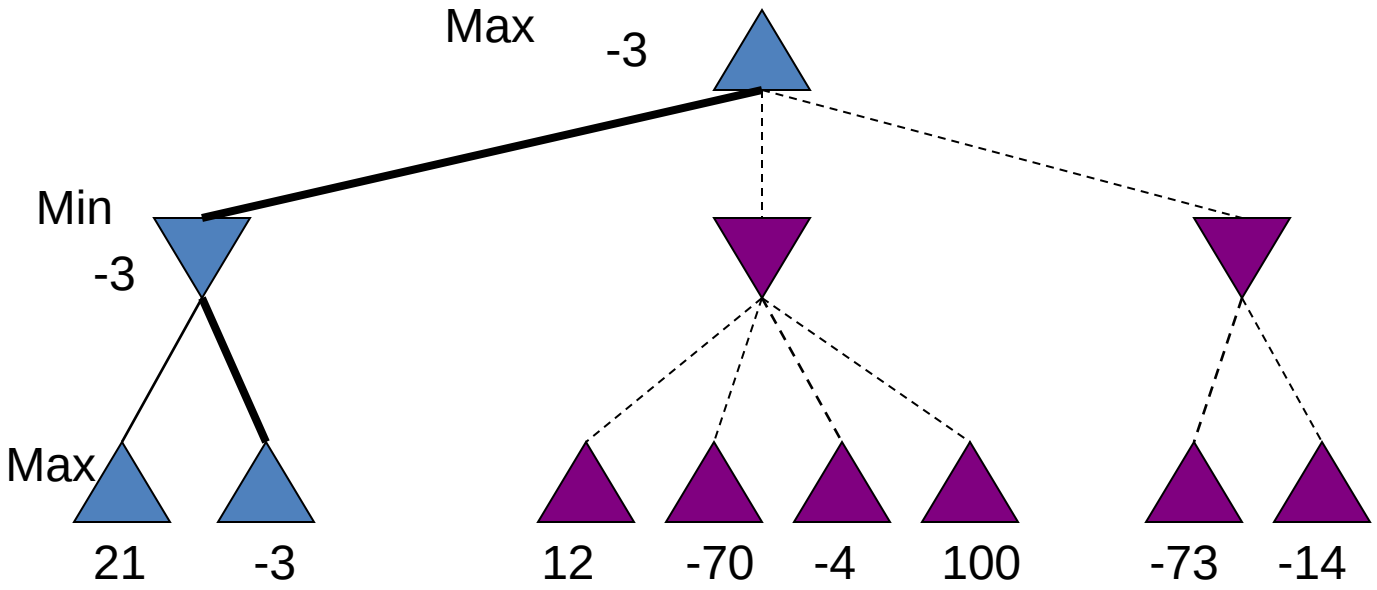
α Cut example



- -3 is minimum so far (second level)
- -3 is maximum so far (top level)

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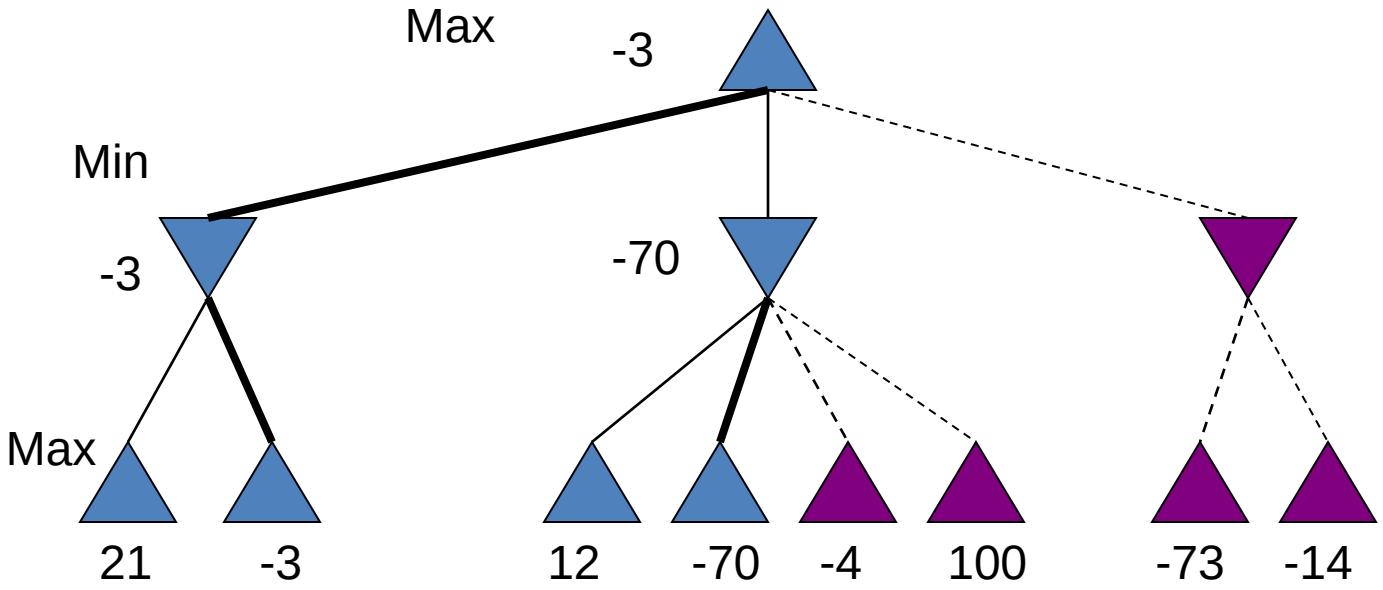
α Cut example



- -70 is now minimum so far (second level)
- -3 is still maximum (can't use second node yet)

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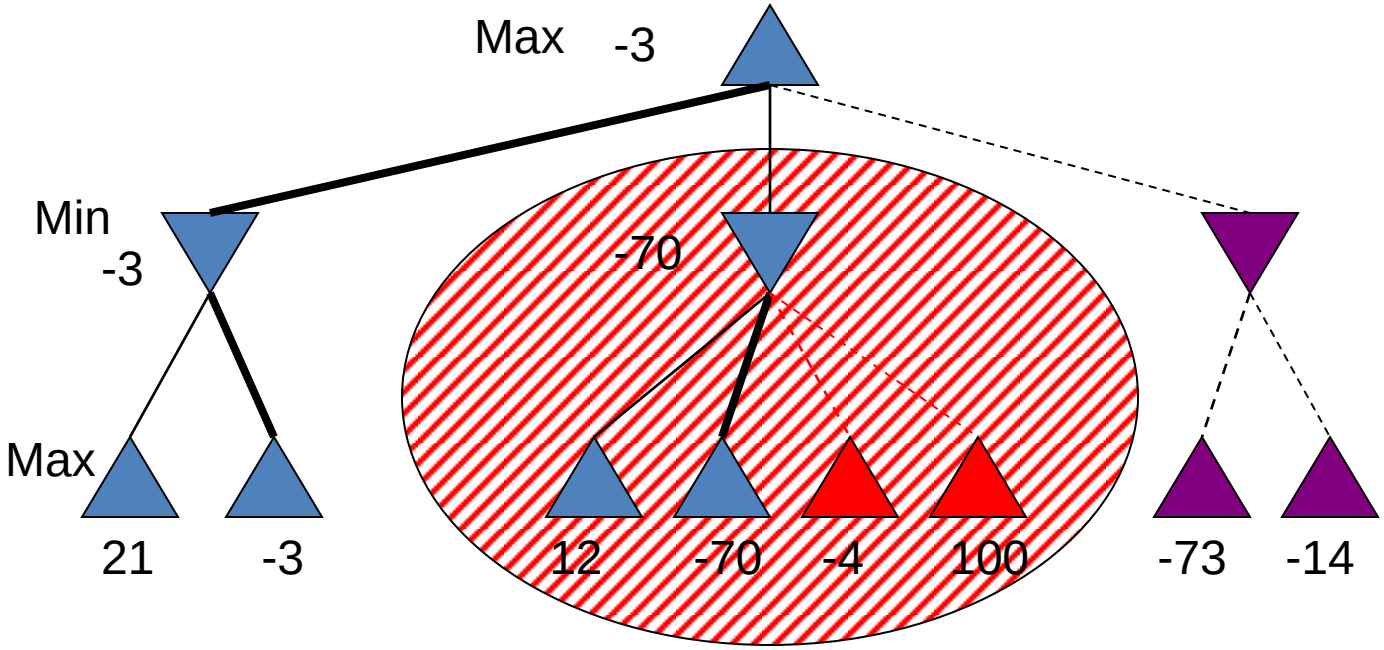
α Cut example



- Since second level node will never be > -70, it will never be chosen by the previous level
- We can stop exploring that node

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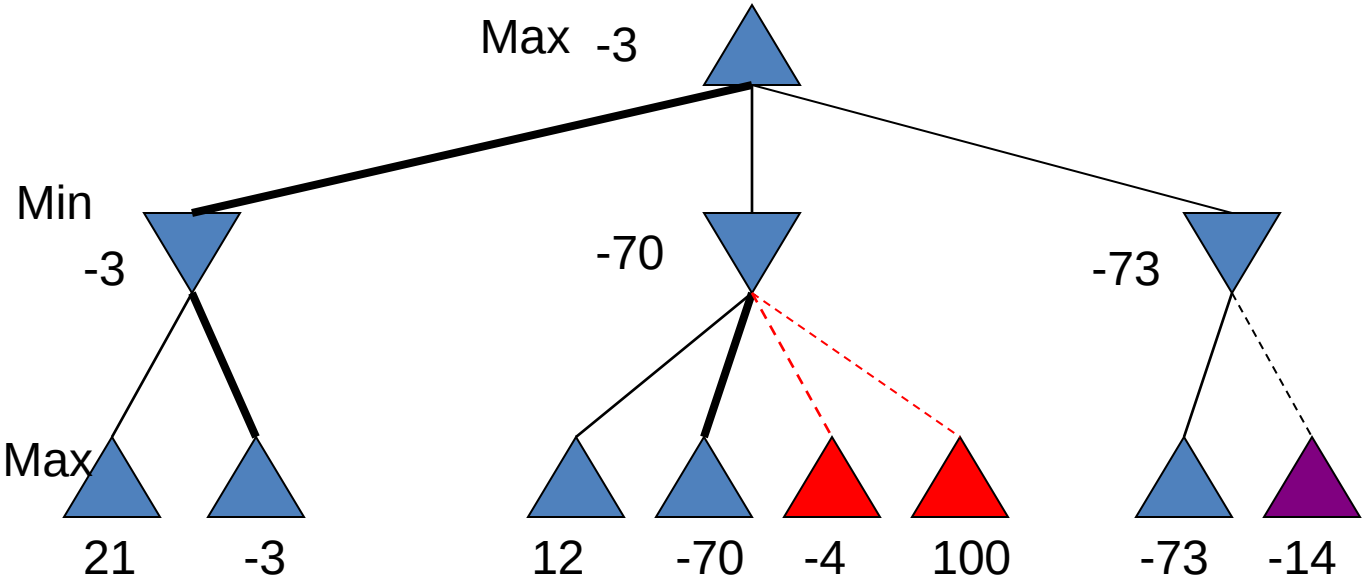
α Cut example



- Evaluation at second level is -73

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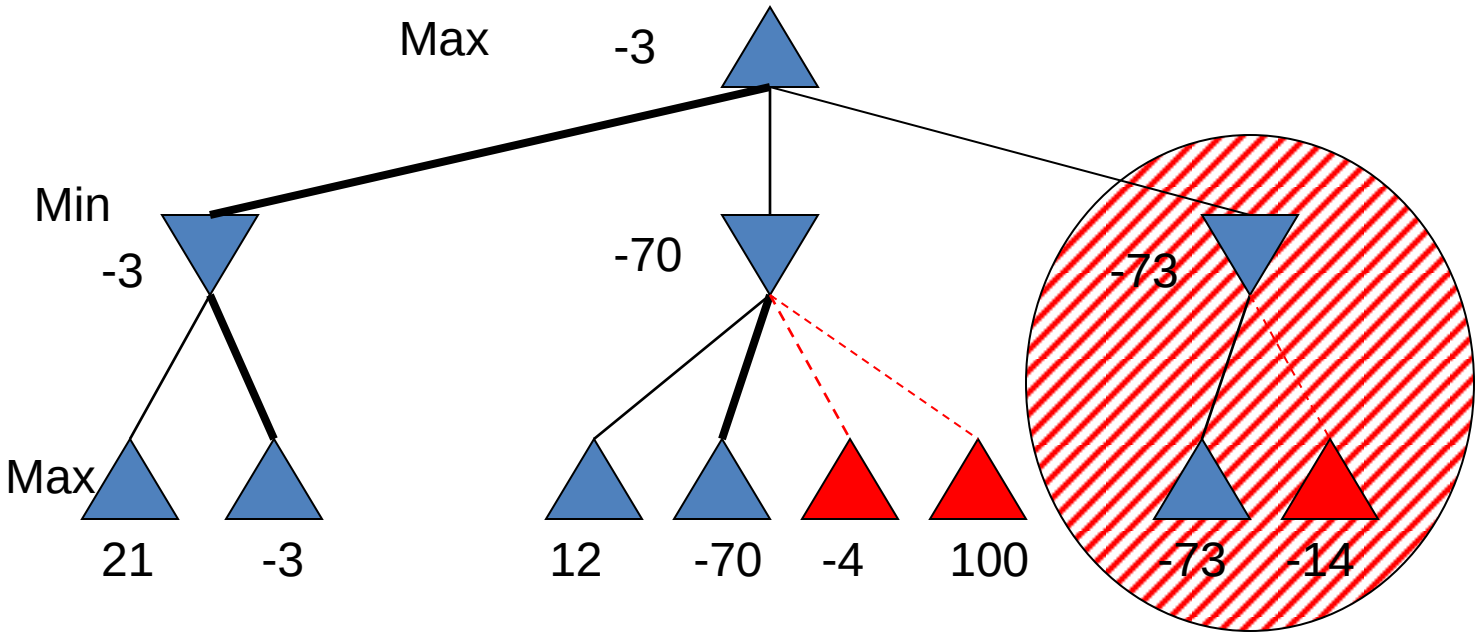
α Cut example



- Again, can apply α cut since the second level node will never be > -73, and thus will never be chosen by the previous level

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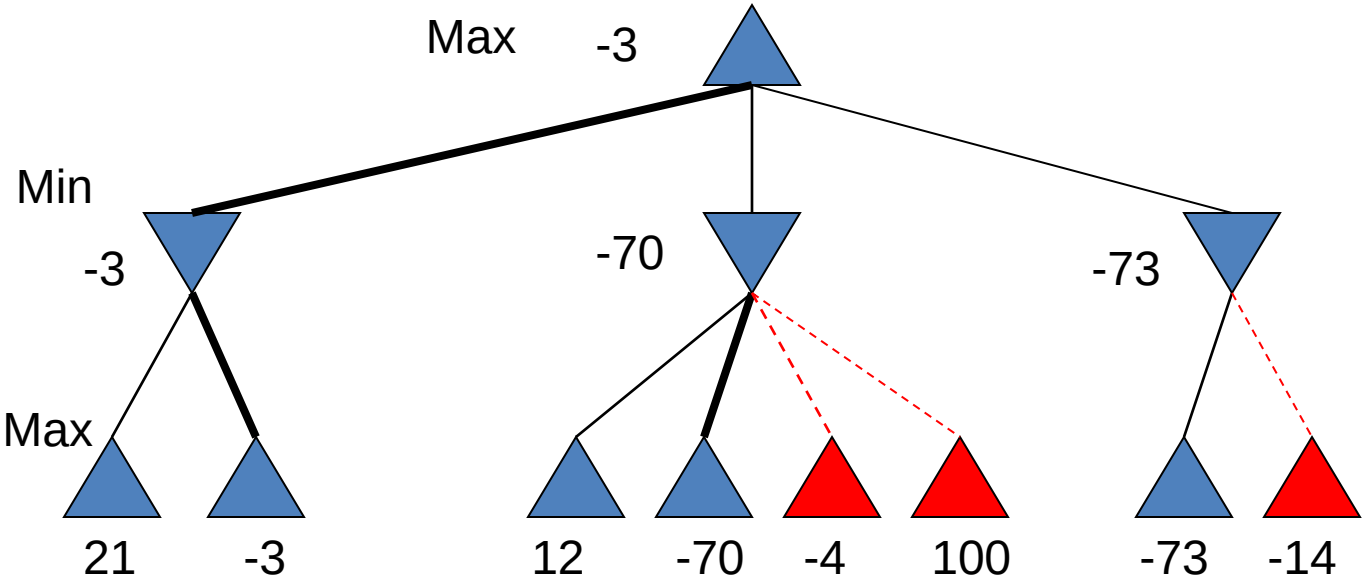
α Cut example



- As a result, we evaluated the Max node without evaluating several of the possible paths

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β cuts

- Similar idea to α cuts, but the other way around
- If the current minimum is less than the successor's max value, don't look down that max tree any more

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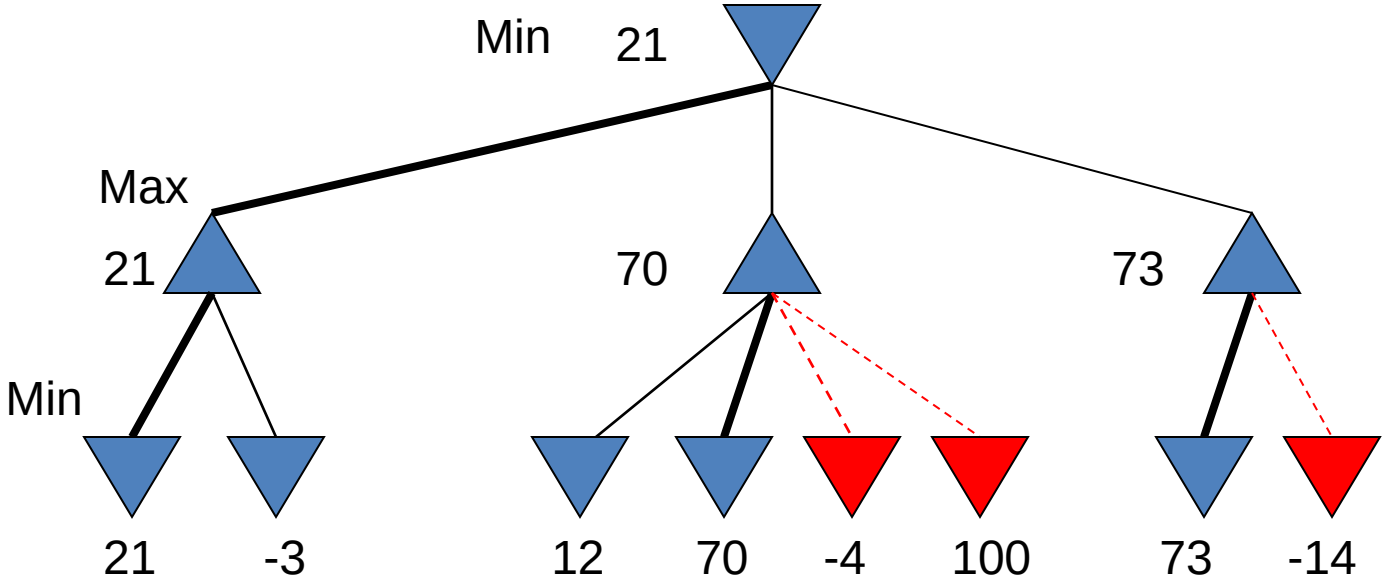
β Cut example



- Some subtrees at second level already have values > min from previous, so we can stop evaluating them.

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α-β Pruning

- Pruning by these cuts does not affect final result
  - May allow you to go much deeper in tree
- "Good" ordering of moves can make this pruning much more efficient
  - Evaluating "best" branch first yields better likelihood of pruning later branches
  - Perfect ordering reduces time to $b^{m/2}$
  - i.e. doubles the depth you can search to!

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α-β Pruning

- Can store information along an entire *path*, not just at most recent levels!
- Keep along the path:
  - α: best MAX value found on this path
    (initialize to most negative utility value)
  - β: best MIN value found on this path
    (initialize to most positive utility value)

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The Alpha and the Beta

- For a leaf, $\alpha = \beta =$ utility
- At a max node:
  - $\alpha =$ largest child utility found so far
  - $\beta = \beta$ of parent
- At a min node:
  - $\alpha = \alpha$ of parent
  - $\beta =$ smallest child utility found so far
- For any node:
  - $\alpha <=$ utility $<= \beta$
  - "If I had to decide now, it would be..."

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Alpha-Beta example

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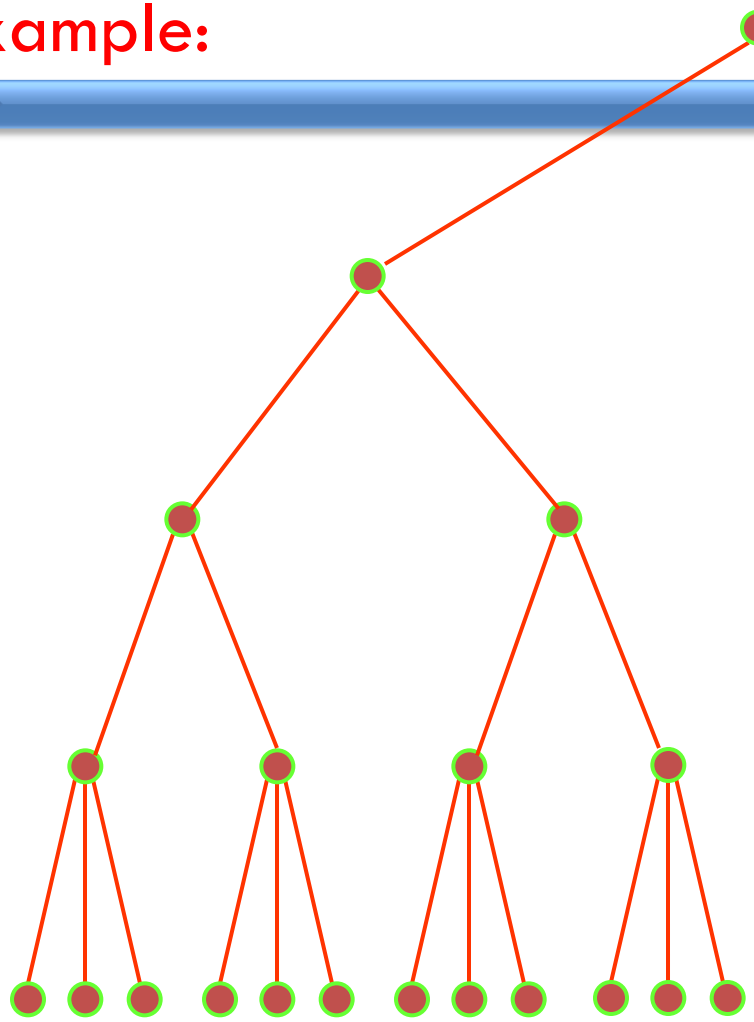
The Alpha-Beta Procedure

Example:



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The Alpha-Beta Procedure

Example:

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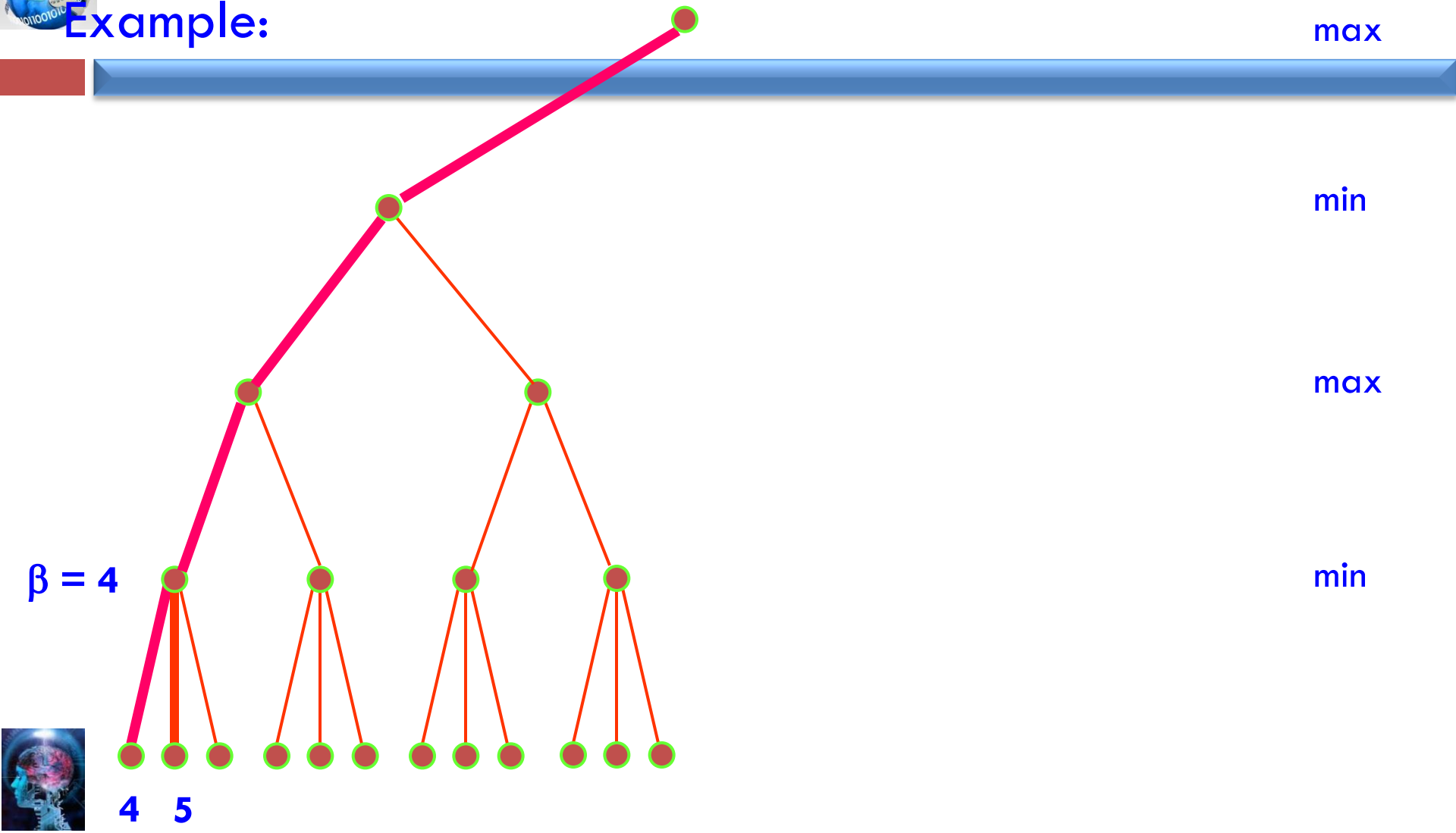
The Alpha-Beta Procedure

Example:



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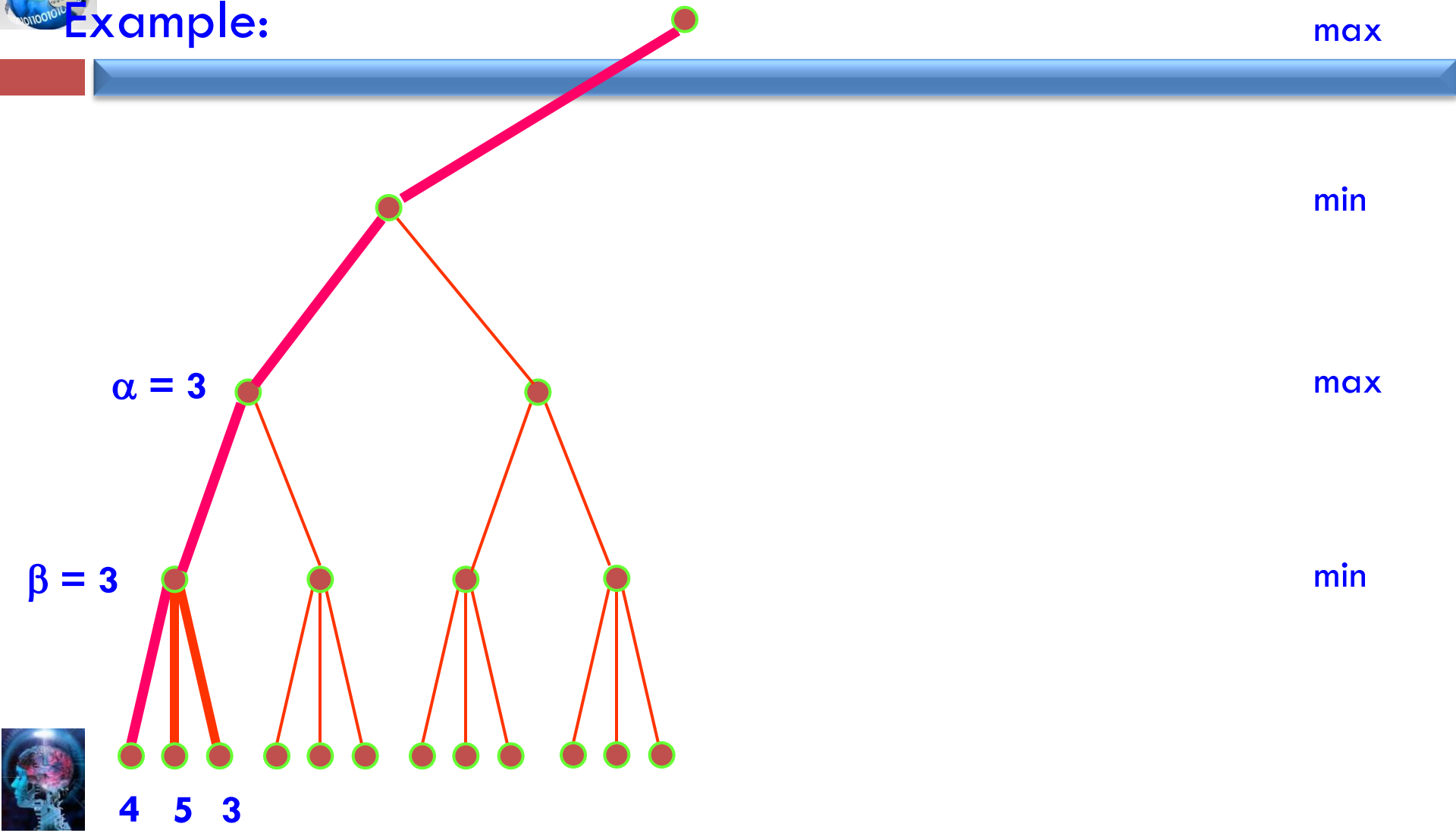
The Alpha-Beta Procedure

Example:



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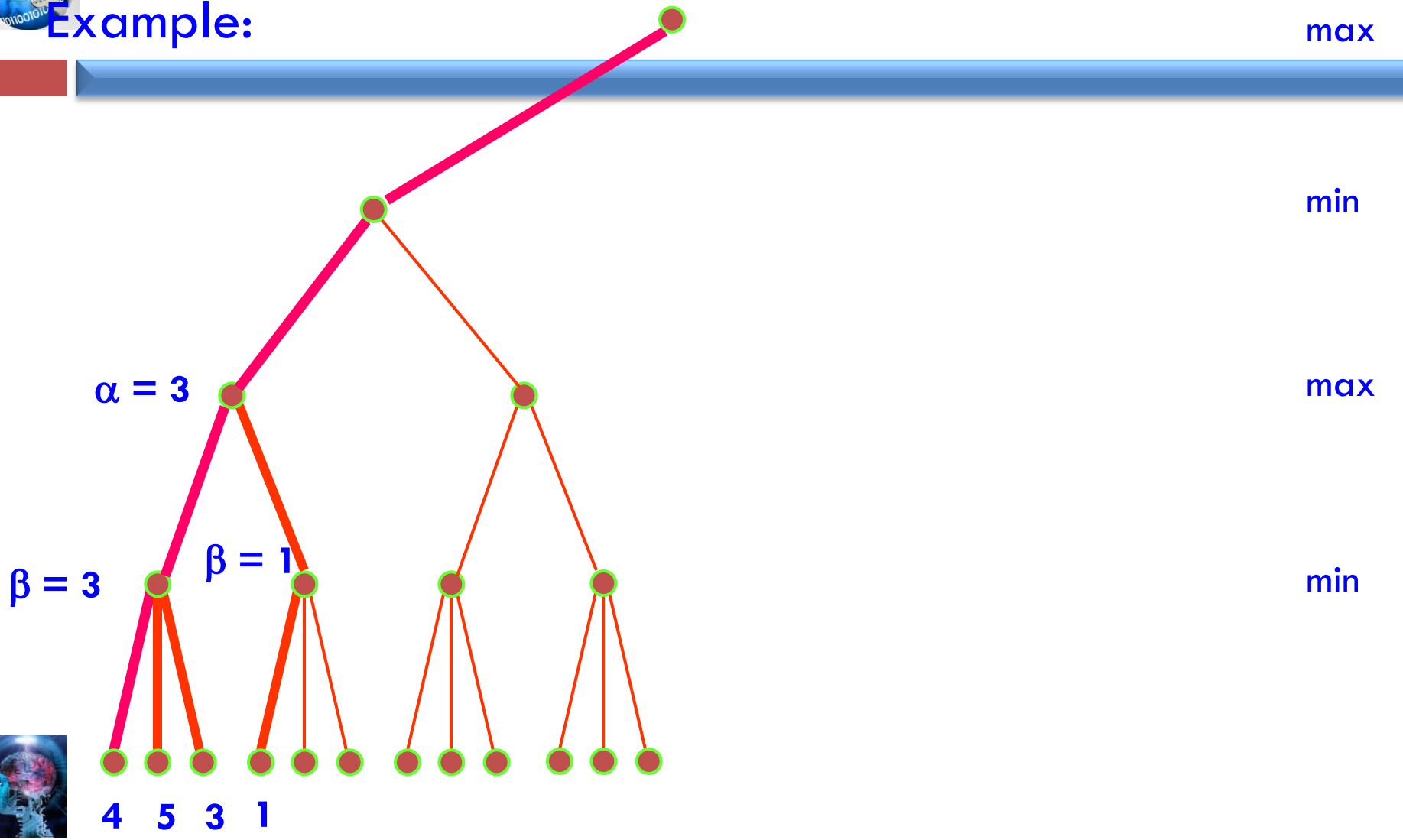
The Alpha-Beta Procedure

Example:



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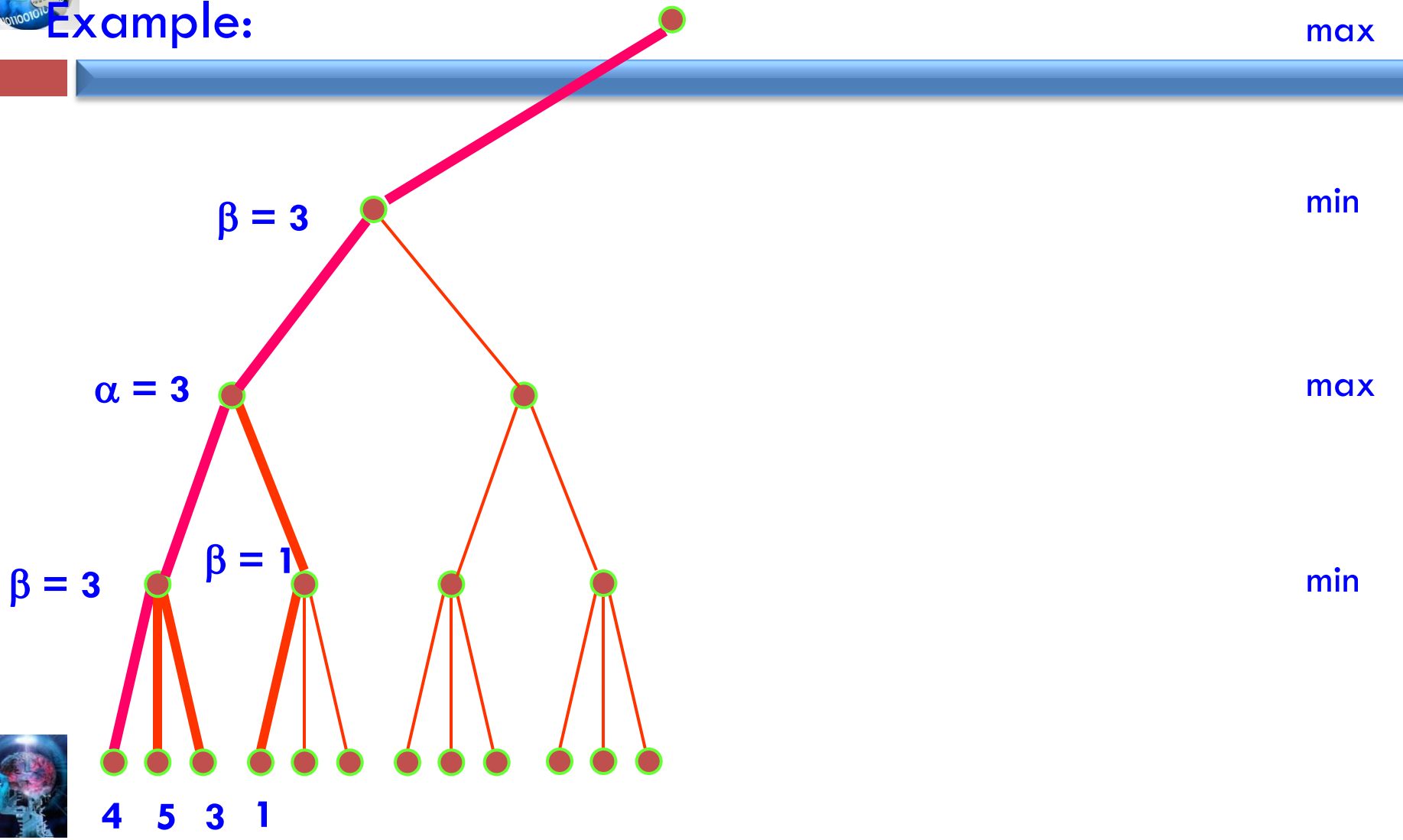
The Alpha-Beta Procedure

Example:



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The Alpha-Beta Procedure

Example:

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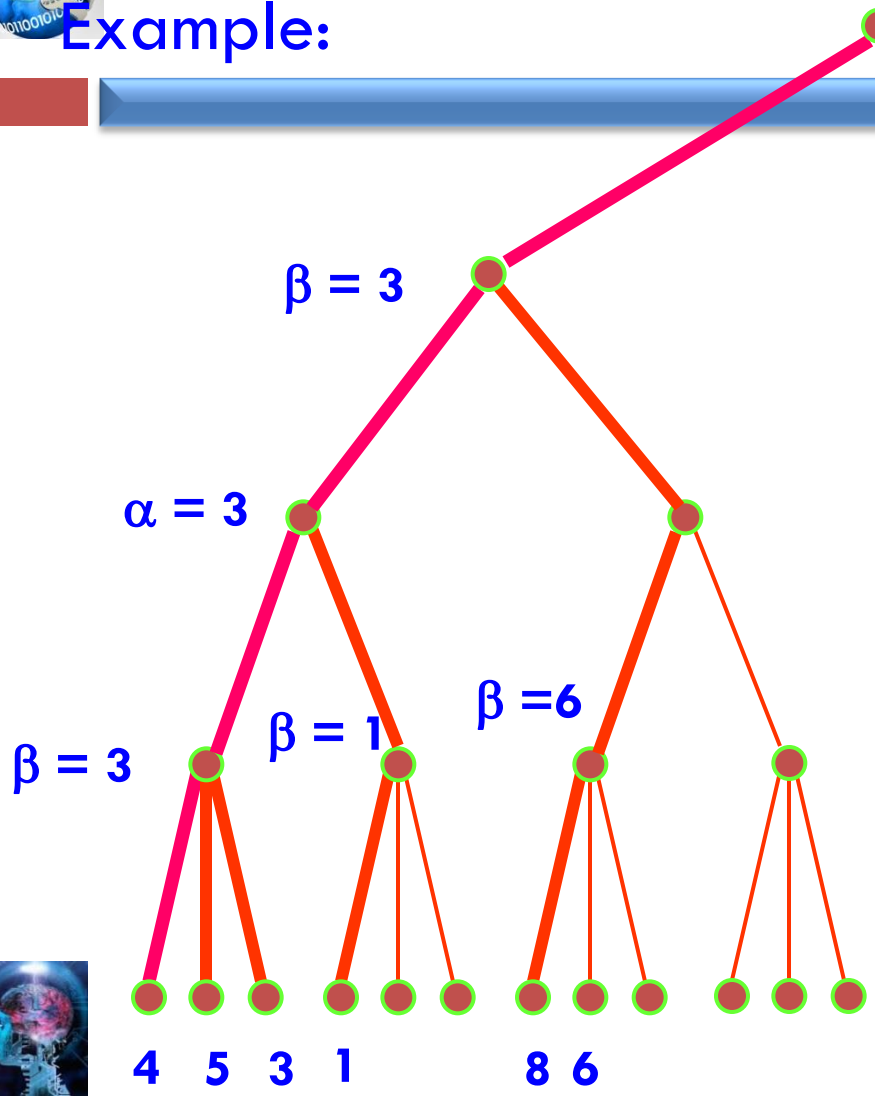
The Alpha-Beta Procedure

Example:



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The Alpha-Beta Procedure

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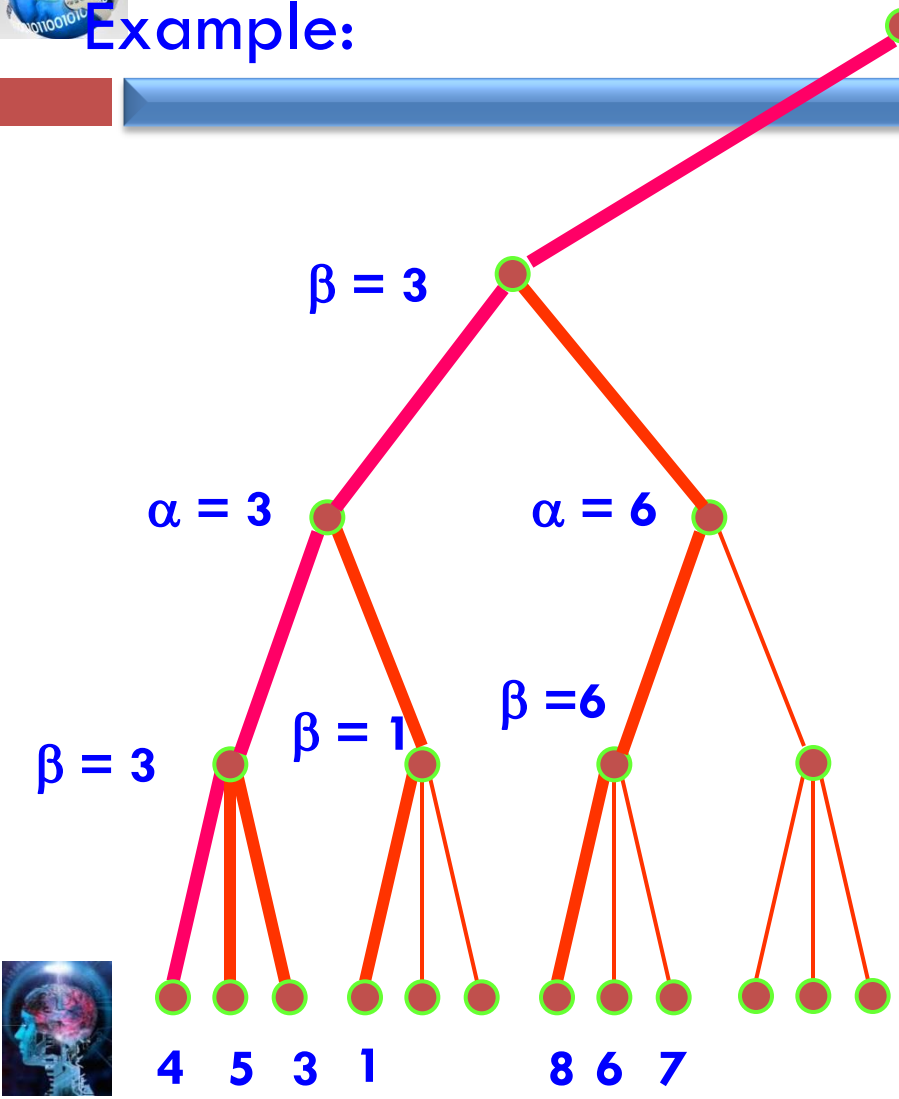

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Thank you

End of Chapter 5

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